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THE UNIVERSITY OF ALBERTA

BASIC INFERENTIAL STATISTICS IN GRADE NINE

by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Basic Inferential Statistics in Grade Nine submitted by Suparak Racha-Intra in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

The purpose of this study was threefold: (1) to construct a unit in basic inferential statistics, (2) to determine the feasibility of using the unit to teach grade nine students, and (3) to investigate the students' anchoring ideas on some selected statistical concepts before and after the instruction of the unit.

The unit in basic inferential statistics in this study was constructed on the basis of the advocates of several reports which promote the instruction of probability and statistics at the secondary school level. The construction of the unit was designed such that it was appropriate for grade nine students. The prerequisites for the unit were kept at a minimal level assuming that general grade nine students would already have attained them. In particular, probability was not a major prerequisite of the unit, rather it was treated as an intuitive experienced prerequisite. The content of the unit was selected with respect to three referenced sources: psychological, social, and mathematical. The emphasis was on the structure of the subject matter. The instructional approach employed in the unit was intuitive in which the intended concepts were illustrated by created examples along with graphical figures and created experiments where appropriate.

The feasibility of using the unit was determined with respect to three areas: (1) the fulfilment of the unit for the five objectives of junior high school mathematics of the Province of Alberta [to develop: (a) understanding of mathematical concepts, (b) skill in the use of fundamental process, (c) the problem solving method, (d) habits of precise thought and expression, and (e) understanding of how to apply the concepts], (2) the student reactions to the unit, and (3) the teacher reactions to the unit.

The data for feasibility assessment were collected by using the developed test instruments, with the exception of the data for the skill in using fundamental processes for which a standardized test was used, and for assessing the development of habits of precise thought and expression for which the data were gathered from classroom observations. The test instrument called "Conception Test" was also developed, using partial interview procedure, to investigate the students' anchoring ideas on some selected statistical concepts before and after the instruction of the unit.

Two grade nine classes with 21 and 27 students from two junior high schools participated in the study. The two classes were taught by the regular school mathematics teachers, using the developed materials of the unit. Instructional times of 18 and 22 forty-minute periods were utilized during March to June 1976.

Upon the completion of the conduct of the study, data were collected and results were analyzed to answer the research questions and test the hypotheses toward the threefold purposes of the study. The major conclusions of the study are: A unit in basic inferential statistics is justifiably constructible such that it is appropriate for grade nine students; The use of the unit is feasible with respect to each of the five objectives of junior high mathematics and also with respect to the overall reactions to the unit on the part of students and teachers; The instruction of the unit caused the improvement of

the students' anchoring ideas on statistical concepts to better quality. On the bases of the results from this study, the inclusion of a unit in basic inferential statistics to a regular school mathematics program at the grade nine level is recommended.

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CHAPTER 1

THE PROBLEM

INTRODUCTION

> . . . There is no disagreement today—nor will there be in the foreseeable future—on the vital importance of mathematics, both to the scientist, engineer, or other specialist called upon to use mathematics in his work, and to the intelligent layman in his everyday life. Mathematical education, to fulfill the needs of an advanced or advancing community, must be under continual scrutiny and undergo constant change, and it is the responsibility of all mathematicians, working in university, school, or industry, to concern themselves with the problem of keeping mathematical education vital and up to date . . . (Cambridge Conference on School Mathematics, 1963)

The above paragraph describes the importance of mathematics, and advocates continual careful examination and change in mathematical education. Because mathematics is a living discipline which covers a wide variety of concepts, it is almost impossible for schools to teach all types of mathematics to their students. The problem is then, what type of mathematics is feasible and worthwhile for the students? According to the report of the Cambridge Conference, probability and statistics are among the recommended areas of study to be included in secondary school curricula.

Statistics is one of the most important and widely used branches of mathematics. It is of great importance for such diverse occupations as government, military, agriculture, insurance, and research. It is also important and useful to the ordinary citizen for his or her effective everyday decision making.

Many statistical concepts are deep and complicated but are particularly useful. Because learning and understanding these concepts requires early familiarization, the Cambridge Conference recommended that they be introduced at the secondary school level. College is rather late for students, especially those who are not mathematics majors, to develop the statistical way of thinking.

Concepts of probability have been introduced in the curriculum of many schools, some as early as the elementary level. The primary purpose of teaching probability, besides its unique mathematical features giving some students intellectual satisfaction, is that it serves as the tool by which students comprehend the uncertainty model of the world. Inferential statistics, a kind of mathematics dealing with the use of a part of a whole in order to get knowledge of the whole, directly involves the knowledge of uncertainty and also makes use of the concept of probability. Inferential statistics, therefore, could follow probability as a sequence in the school curriculum. Moreover, the concepts of descriptive statistics would be automatically included, since descriptive statistics is a part of inferential statistics.

Inferential statistics can be approached in an intuitive fashion using examples, experiments, and graphs. The concepts intended to be taught are still valid and useful to the students, and if properly taught should be exciting and interesting as well.

Because inferential statistical procedures are useful to everybody, all students should have a chance to learn them before going out into society. Grade nine should be a suitable level for this purpose since grade nine students, having passed two years of mathematics at the secondary school level, should have sufficient prerequisites and should have attained acceptable skills in the use of fundamental mathematical processes in order to study inferential statistics.

Have the schools prepared and provided such mathematical studies for students? In practice, statistics has not been focused upon in any particular grades. It has been taught to the students without any particular time allotment in the curricula. The emphasis is more on the mathematical manipulation of formulas, symbols, and numerical data, rather than on basic statistical concepts. The National Advisory Committee on Mathematical Education, in their report, Overview and Analysis of School Mathematics Grades K-12 (1975), has summed up the instruction of probability and statistics in the schools as follows:

> . . . Many recent curriculum planning conferences and resultant development projects have given prominent attention to statistics and probability throughout elementary and secondary mathematics programs. While probability instruction seems to have made some progress, statistics instruction has yet to get off the ground. At the elementary school level, most common topics are only traditional graphing exercises and elementary descriptive statistics. Furthermore the NCTM exploratory survey indicates that these topics get very little time in the average teacher's mathematics instruction. At the high school level probability topics . . . are commonly omitted. A one semester senior course in probability and statistics has gained only small audiences of the very best students. Furthermore, this course places a heavy emphasis on probability theory, with statistics, if treated at all, viewed as merely an application of that theory . . . (p. 45)

This study is primarily concerned with the development of an instructional unit in basic inferential statistics, the feasibility of teaching the unit to grade nine students, and the investigation of the students' ideas about some statistical concepts.

NEED FOR THE STUDY

Although there have been some previous studies concerning the introduction of teaching probability and statistics at the secondary school level, such as those by Smith (1966), Shulte (1970), and others, there still exists a need for further study for the following reasons:

1. Probability and statistics is a large field. Different studies of different topics have drawn inferences which may not be valid for the whole field. Most previous studies emphasized the concepts of probability and some concepts of descriptive statistics. This study, on the other hand, consists of a sequence of topics forming a coherent unit which emphasizes the basic concepts of inferential statistics, that is, the idea of getting information about a total (a population) by means of its fraction (a sample).

2. Different studies have employed differing approaches and teaching strategies, and since the quality of instruction is one of the major variables in any study, the differences in outcomes of those studies may be due to variations in the quality of instruction. This particular study employs the intuitive approach, in which concepts are illustrated by the use of examples and experiments.

3. The result of the instruction of a unit in probability and statistics to grade nine students, according to the study by Shulte (1970), was a decline in student attitude and student skill in the

use of division. These aspects of study need reinvestigation in a different population, context, instructional approach, and environment.

4. The investigation of the students' existing ideas about some statistical concepts, in particular the students' statistical knowledge obtained outside school, has not been conducted before. The students' prior knowledge about statistical concepts will greatly influence their learning. Ausubel (1968) says, "The most important single factor influencing learning is what the learner already knows."

Therefore, this aspect of learning, particularly as it relates to the learning of statistical concepts, needs further investigation. The results will be useful in the instruction of statistics, the construction of a unit, and the assessment of a unit's instruction.

THEORETICAL BACKGROUND

The main theme of this study involves two aspects: the first one is the introducing of the subject—inferential statistics, which is not normally taught at the secondary school level; the second is the exploring of the students' ideas about some of those statistical concepts to be taught. Bruner and Ausubel provide the theoretical background which initiated, and which supports, these goals.

> . . . Any subject can be taught effectively in some
> intellectually honest form to any child at any stage
> of development . . . (Bruner, 1960)

Bruner has suggested the modification of the conception of readiness to include not only the child but the subject matter as well. Subject matter can also be modified and passed through stages of readiness. One can view the development of a child as evolving through stages in which his preferred modes of representation are serially: enactive, ikonic, and symbolic. Similarly, the basic principles or the structure of inferential statistics can be represented manipulatively, as visual representations, or as formal symbolic expressions according to their suitability to the learner's stage of development. In relation to the above assertion, Bruner has suggested that, first, the fundamental principles or structures of disciplines are essentially simple; and second, or consequently, that these simple structures can be taught and learned in an intellectually honest form through some mode of representation. Bruner's statement relates to the first aspect of this study in that the construction of a unit in basic inferential statistics can be viewed as the modification of this subject matter to be appropriate for grade nine student readiness.

> . . . If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly . . . (Ausubel, 1968)

According to Ausubel, every learner has a cognitive structure, which refers solely to the stability, clarity, and organization of a learner's subject-matter knowledge in a given discipline. The actual ideas and information embodied in this knowledge are cognitive content. The existing ideas or background with respect to a given discipline, available to a learner in his cognitive structure at any stage of time, are referred to as anchoring ideas. Meaningful learning, the most efficient human mechanism for acquiring and storing new ideas and information in a given discipline, will occur due essentially to the presentation of new suitable learning material to a learner. Furthermore, that learner must possess the necessary relevant anchoring ideas in his cognitive structure to which the new learning material can be related in a certain appropriate fashion. If those relevant anchoring ideas are not available to him in his cognitive structure, then rote learning will take place instead.

In relation to the second aspect of this study, one can presume that the students, before studying the unit in statistics, should have some anchoring ideas about statistical concepts in their cognitive structures. Once the lessons on statistics, viewed as a set of new learning materials, are presented to them, the learning outcome is essentially determined by the availability of prior anchoring ideas. Also, once the students complete studying the unit in statistics they will have some form of anchoring ideas about those statistical concepts available in their cognitive structures for later instances of use.

THE PROBLEM

It is the threefold purpose of this study (1) to construct a unit in basic inferential statistics, (2) to determine the feasibility of teaching the unit to grade nine students, and (3) to investigate the existence and change in the students' anchoring ideas about some statistical concepts before and after the unit instruction.

Answers to the following questions were sought:

1. Can a unit in basic inferential statistics be constructed

in such a way that it is appropriate for a grade nine student population, and so that it fulfills the five junior high school mathematics objectives of Alberta listed below?

a. To develop an understanding of mathematical concepts and appreciation of mathematical structure.

b. To develop skill in the use of fundamental processes.

c. To develop systematic methods of analyzing problems and presenting their solutions.

d. To develop habits of precise thought and expression.

e. To develop an understanding of the significance and application of mathematics in the modern world.

2. After the unit in basic inferential statistics has been constructed, is its use feasible in grade nine? The feasibility of using the unit is determined with respect to the assessment in three areas.

2.1 How well does the unit contribute to the Alberta curriculum, in particular, the five objectives of junior high mathematics? In other words, can grade nine students who study the unit adequately learn the statistical concepts with respect to each of the five objectives?

2.2 How do grade nine students react to the unit in basic inferential statistics? Specifically, how do the students react to the unit in the following areas: (a) enjoyment of activities, (b) difficulty of the subject, (c) interest of the content, (d) belief of having new knowledge gained, and (e) perceived usefulness of statistics?

2.3 What are the teacher reactions to the unit in basic inferential statistics? Specifically, what opinions do the teachers who teach the unit hold in the following areas: (a) perception of the unit's appropriateness in relation to the general five objectives of junior high mathematics, (b) perception of the instructional suitability, (c) perception of demands made on the teachers' subject background in teaching the unit, (d) perception of student reactions to the unit, and (e) perception of the unit's being included in a regular mathematics program.

3. The investigation of the students' anchoring ideas.

3.1 What are the students' anchoring ideas about some selected statistical concepts before and after the instruction of the unit in basic inferential statistics?

3.2 How do the students change their anchoring ideas about some selected statistical concepts after the unit instruction?

DEFINITION OF TERMS

Inferential statistics is a subject which deals with the use of numbers derived from a set of data (a sample) to give numerical information about a larger set of data (a population) from which the original set of data (the sample) was taken.

Descriptive statistics is a subject which deals with the use of numbers derived from a set of data in order to summarize information about that set of data.

Feasibility is the capability of being put into effect with respect to some criteria of judgement in certain referenced domains. In this study, the capability of being put into effect is measured with respect to specified criteria of judgement in three referenced domains: the unit contribution to the five objectives of junior high mathematics, the student reactions and the teacher reactions.

A concept is one of the following: (Merrill and Wood, 1974)

1. A symbol, object, or event associated on a one to one basis with another symbol, object, or event. For example: Σ is a symbol for summation, and $\bar{x}$ is a symbol for a sample mean.
2. A set of objects, symbols, or events (referents) which have been grouped together because they share some common characteristics. For example: "sample" is a set whose elements consist of all non-empty subsets of a certain population.

Appropriateness is the relevance of something with respect to one's justification according to a referenced model. In this study, the unit constructibility is justified according to its relevance to a grade nine student population and the five objectives of junior high mathematics.

To adequately learn is to be able to express the learning of a certain specified concept in a way which satisfies a certain criterion level of judgement. Operationally, it is to score on the achievement test high enough to reach or exceed the criterion referenced measurement of the test.

To react is to express an opinion on something with respect to a certain variable or aspect of that thing. Operationally, it is to express the degree of agreement to a given statement about the unit on a five point scale.

An anchoring idea is a learner's knowledge of a certain concept or a given subject matter which is available to him in his cognitive structure at a certain stage of time.

The unit is the entire constructed statistical program to be used during the course of study.

OUTLINE OF THE REPORT

In the present chapter the problem of the study has been introduced. The literature related to the study is reviewed in Chapter 2. The construction of the instructional unit is described in Chapter 3. The instruments employed in this study are described in Chapter 4. The description of the design and conduct of the study appears in Chapter 5. The results of the study are reported in Chapter 6. The summary, conclusions, and recommendations are included in Chapter 7.

CHAPTER 2

REVIEW OF LITERATURE

INTRODUCTION

The review of literature relating to this study is divided into three main sections. The first section is devoted to the various influential reports that advocate the inclusion of probability and statistics in the school curriculum, followed by a review of the school mathematics program in Alberta. The second section is a review of what has been done about some related curriculum research on probability and statistics. And the third section deals with some learning theories relating to the instructional approach employed in this study and to the preparation of materials for the instructional unit.

The purpose of the third section is mainly to establish the theoretical foundation for the construction of the unit described in Chapter 3. However, this section is not included as a part of that chapter because it also deals with the notion of anchoring ideas which is basic to the theoretical background behind this study as a whole, in particular, the third purpose of the study.

ADVOCATES OF STATISTICAL IDEAS IN SCHOOL CURRICULA

There are several reports that promote the instruction of probability and statistics at the secondary school level, and the recommendations made by these reports have influenced school mathematics both in content and approach. These reports are: (1) the College Entrance Examination Board (1959), (2) the Cambridge Conference on School Mathematics (1963), (3) the Committee on the Undergraduate Program in Mathematics (Statistics) (1972), (4) the National Advisory Committee on Mathematical Education (1975), (5) the School Mathematics Project (1961), (6) the Oxford Middle School Mathematics (1971), and (7) the Alberta Mathematics Program of Studies.

College Entrance Examination Board (1959)

The report of the Commission of Mathematics on behalf of the College Entrance Examination Board, entitled Program for College Preparatory Mathematics, has as its primary goal the improvement of the mathematics program for college capable students. It presents a nine point program for such students, one of which is the recommendation that alternative units be added for grade 12—either introductory probability with statistical application, or an introduction to modern algebra. The two main objectives of the probability and statistics course are: (1) to introduce the student to probability concepts and to the mathematics involved in these ideas, and (2) to illustrate ways in which these concepts apply to certain common statistical programs.

The above course on probability and statistics for grade 12 is considered as "advanced mathematics" according to the report. Along this line, as the prerequisite mathematics and the intermediate mathematics, the report recommends two basic courses: (1) Algebra and Statistics for Grades 7 and 8, and (2) An Introduction to Statistical Thinking for Grades 9-11. The content of statistics in the first course emphasizes reading and constructing various graphs, such as the reading and construction of bar graphs, line graphs, pictograms, circle graphs, and continuous line graphs. The content of the second course, on the other hand, emphasizes descriptive statistics which includes: (1) Statistical data; sets of observations and measurements, (2) Collection and organization of data (charts, tables, graphs), (3) Use of single numbers to characterize a set of data; average (median, mean), and (4) Simple measures of dispersion; range and quantities.

The outline of probability and statistics being recommended by the report to be included in the secondary school curriculum is roughly divided into three phases: (1) to have students get used to reading and constructing various basic statistical graphs in grades 7 and 8, (2) to introduce some concepts of descriptive statistics to students in grades 9-11, then (3) the more formal course on probability and statistical application is introduced in grade 12. In conclusion, the area of descriptive statistics is introduced first, then probability is introduced, followed by statistical application which can be viewed as inferential statistics—a combination of both descriptive statistics and probability.

The report, despite its primary concerns about the program for college capable students, also recognizes the importance of mathematics in general education, which involves the majority of students who are not bound for college. Two of the objectives of mathematics in general education proposed by the commission emphasize the inclusion of statistics and appreciation of mathematics:

> 1. An understanding of, and competence in, the process of arithmetic and the use of formulas in elementary algebra. A basic knowledge of graphical methods and simple statistics is also important. . . .
>
> 4. An understanding of mathematics as a continuing creative endeavor with aesthetic values similar to those found in art and music. In particular, it should be made clear that mathematics is a living subject, not one that has long since been embalmed in textbooks. . . . (p. 11)

Cambridge Conference (1963)

In the summer of 1963, a group of twenty-five professional mathematicians and mathematics users held a conference on school mathematics in Cambridge, Massachusetts. The main purpose of the conference was to review school mathematics and establish tentative goals for mathematical education, including considerable detail such as the curriculum content for grades K-12.

Some of the broad goals for the school mathematics curriculum presented by the conference are:

1. The subject matter through the full thirteen years of study in grades K-12 should have a level of training equivalent to three years of the present top level college, which is equivalent to two years of calculus and one semester each of modern algebra and probability theory.

2. Acquisition of some skills may be reduced or abandoned, especially in the case of drill for drill's sake, because the proposal for more content in the curriculum must be compensated by the omission of something else, such as skill practice. Instead, the conference recommended the replacement of unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts. They believed that entirely adequate technical practice can be woven

into the acquisition of new concepts. However, the conference did recognize some drill problems for the individual student whose technical skill is behind. Such drills should be, however, toward the acquisition of new concepts. In their words:

> We are definitely opposed to the view that the main objective is arithmetic proficiency and that new, interesting concepts are being introduced primarily to sugar-coat the bitter pill of computational practice. . . . (p. 8)

3. The students should have early familiarity with mathematics. The conference set a broad goal toward this end that: the elementary school program should be understandable to virtually all students and lead to a level of competence well above that of the general population today. As fewer and fewer students are expected to elect mathematics in junior and senior high schools, the attempt has to be made first to provide the students early familiarity with suitable mathematics. Of particular importance is an elementary feeling for probability and statistics.

4. The broad goal toward pure and applied mathematics should be compromised. The conference remarked that there will be more users of mathematics than makers. The conference also recognized the justification of fears concerning the early introduction of rigorous mathematics—that the students would lose interest in less precise disciplines, and of rigid mathematics, which requires proof of every small point in the math course—that the students would be able to do nothing in a typical applied situation. To this end the conference recommended the logical rigor procedure, the use of logical precision in an intuitive way as in their own words:

> . . . If the nature and limitations of the mathematical models used in science are carefully described and if the intuitive steps which go into their construction are fairly presented, then the inherent attraction of unlocking the secrets of nature or solving a practical problem should easily balance the attraction of logical certitude.
>
> To foster the proper attitude toward both pure and applied mathematics we recommend that each topic should be approached intuitively, indeed through as many different intuitive considerations as possible. . . . (p. 11)

To facilitate reaching these broad goals, the conference also suggested some pedagogical principles and techniques. Among them is the teaching of new topics by the intuitive fashion.

> . . . At the first stage an intuitive or premathematics approach offers the opportunities of an early introduction of important concepts. There is time for each of these concepts, first drawn from the student's general experience, to be made more familiar and more precise, and time to develop the concept further. The concept can be used by the student from the beginning in appropriate simple contexts. (pp. 13-14)

The conference suggested that probability and statistics, in particular, be taught in four doses through the curriculum as follows (p. 71):

1. In the elementary school, empirical study of the statistics of repeated chance events, coupled with some arithmetic study of the working of the law of large numbers.

2. In junior high school, probability as an additive set function on finite sets, conditional probability, independence, binomial distribution, expectation, variance, and some simple statistical tests.

3. In senior high school, after the first work on limits and series, probability as an additive set function on countable sets,

Poisson distribution, law of large numbers, etc.

4. In senior high school, after integral calculus, probability as an additive set function of intervals on the line, continuous distributions on the line and in several dimensions, normal distribution, limit theorem, etc.

In the curriculum for grades 7-12, the conference proposed two topical outlines. The probability and statistics courses for junior high school, grades 7 and 8 in particular, consist of (pp. 43, 45): (a) binomial theorem, combinational problems, (b) review of earlier experience with probability, basic definitions in probability theory for finite sample spaces, (c) sampling from a finite population, unordered sampling, ordered sampling with and without replacement, (d) conditional probability, independence, (e) random variables and their distributions, (f) expectation and variance, Chebychev's inequality, (g) joint distribution of random variables and independent variables, (h) Poisson distribution, and (i) statistical estimation and hypothesis testing. The content of the two topical outlines is almost identical except that one does not have binomial theorem, combinational problems.

In conclusion, probability and statistics, according to the proposal of the conference, should be closely tied together all the way through the school mathematics curriculum, and statistics concepts could be viewed as the illustration or application of probability concepts.

Committee on the Undergraduate Program in Mathematics (Statistics) (1972)

The report by the panel on statistics is entitled Introductory Statistics Without Calculus. The panel's primary goal was to recommend an efficient course of instruction for nonmathematics majored students. Although the report aims at instructional improvement at the college level, its recommendations are worth taking into account at the secondary school level. These college students do not have a calculus prerequisite; it is the first statistics course for them and also the terminal course for most of them. The situation is not much different then, from the majority of students in secondary schools.

The report starts out by citing criticisms of introductory statistics courses made by the people involved. Some of these criticisms are (p.2): (1) the emphasis is too mathematical or probabilistic without providing sufficient insights into statistical concepts, (2) the course is too technique oriented, overemphasizing computation and underemphasizing the fundamental ideas underlying statistical reasoning, and (3) insufficient attention is paid to drawing statistical inferences from real data.

To counter some of these criticisms, the following guidelines for formulating course objectives are recommended by the report (p. 3): (a) the course must have limited objectives; otherwise, it is likely none of them will be met adequately, (b) the primary objective of the course should be to introduce students to variability and uncertainty and to some common concepts of statistics, and (c) a secondary objective of the course should be to teach the student some common statistical formulas and terms and some of the widely used statistical techniques.

The report presents the implications of the above recommended objectives as follows (p. 4): (a) proofs and extensive manipulations on formulas should be employed sparingly, (b) the course should not dwell on computational techniques, rather, some electronic calculators are recommended, (c) probability concepts are important but should not constitute the dominant portion of the course, and (d) to illustrate the application, the course must be data oriented and must incorporate analysis of real world data.

In conclusion, the report strongly recommends more emphasis on inferential concepts and data analysis, and less on mathematical elements.

National Advisory Committee on Mathematical Education (1975)

The National Advisory Committee on Mathematical Education (NACOME) presented a technical report, Overview and Analysis of School Mathematics Grade K-12, in attempting to assemble a comprehensive overview and analysis of the current status of mathematics education—particularly its objectives, current and innovative practices, and attainments.

According to NACOME, statistics and probability, despite the prominent attention of many curriculum planning conferences and resultant development projects, have not made much progress in the schools, in particular statistics: "While probability instruction seems to have made some progress, statistics instruction has yet to get off the ground . . . (p. 45)." At the elementary school level, the most common topics are graphing exercises and elementary descriptive statistics, and these topics get very little time in the average teacher's instruction. At the high school level, probability and statistics are commonly omitted, and only a handful of the very best students are involved in the senior course. Moreover, this course places a heavy emphasis on probability theory, while statistics, if treated at all, is only the application of that theory.

NACOME also reported on what has been done to encourage the inclusion of statistics in the school curriculum.

In 1967, the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) sponsored the Joint Committee on the Curriculum in Statistics and Probability. This committee focused its efforts on two main tasks: (1) to persuade school boards, principals, teachers, and parents of the importance and usefulness of statistics, and that it should become a part of the school curriculum, (2) to prepare materials for the teachers' use and consultation.

To accomplish the first task, the committee prepared a volume of essays, Statistics: A Guide to the Unknown, in which famous statisticians described in nontechnical language important applications of statistics and probability. The content of the book involves a variety of statistical applications relating to man in his biological world, man in his political world, man in his social world, man in his physical world and others.

When proceeding with the second task, the committee found no shortage of good probability texts at the secondary school level. On the other hand, statistics materials were hard to find at the pre-college level. Because it is unrealistic to expect teachers to assemble appropriate teaching materials on their own, the committee produced four booklets entitled Statistics by Examples, in which 52 real life problems with real data are explored in great detail, extending from data organization to sophisticated model building. Each problem represents a series of learning experiences which also includes additional exercises and projects.

Since 1974, and continuing into 1976, the Joint Committee has conducted working sessions to acquaint teachers with some of the materials, but much more still needs to be done. Sales of the book, Statistics: A Guide to the Unknown, have reached about fifty thousand copies, but the majority have gone to college students, not the intended public. Sales of the series, Statistics by Examples, have reached about fifteen thousand copies, but again, the teachers who have tried the series still indicate a need for more elementary resource material and specific guidelines on where and how to use statistical topics in normal curricula. In conclusion, NACOME states:

> Development of these materials should have high priority in curriculum making. But most important is acceptance of the premise that statistics can and should be taught all the way from kindergarten through grade 12. . . . (p. 47)

The following are recommendations made by NACOME for dealing with statistical ideas in the school curriculum (pp. 47-48):

1. Use statistical topics to illustrate and motivate mathematics.

2. Emphasize statistics as an interdisciplinary subject with applications in the natural, physical and social sciences, and in humanities. Possibilities for interdisciplinary courses are: a course oriented towards computers and statistics, and courses in physical, biological and social sciences using statistical tools.

3. Develop several separate courses dealing with statistics to meet varied local conditions. Two possible courses are:

a. A statistics course for high school students with little or no algebra, especially noncollege bound students or college bound students in social sciences who, as consumers and citizens, must learn to cope with numerical information. The main theme of such a course would be "making sense out of numbers" without getting involved in complicated mathematical formulas.

b. A senior year statistics course with a probability prerequisite for more mathematically and scientifically minded students. The need for such a course would be greatly enhanced if CEEB introduced an Advanced Placement Program in Statistics, and by the inclusion of a substantial number of statistical problems on standardized tests.

School Mathematics Project (SMP)

This project was founded in 1961. The main objective was to devise radically new mathematics courses which would reflect, more adequately than did the traditional mathematics, the up-to-date nature and usages of mathematics. The team of writers and researchers for the project, including several school and university mathematicians, completed the main series of Book 1-5 forms of pupils texts, starting at age 11+ and leading to the O-level examination in mathematics; and the advanced mathematics, Books 1-4 covering the curriculum for A-level examination in mathematics.

By 1967, the project found that the mathematical content in the SMP texts was suitable for a much wider range of pupils than originally anticipated, especially in comprehensive schools, but that the presentation of the content needed adaptation. As a result, a new series, Books A-H, was produced in order to serve as a secondary school course starting at age 11+. These books are specially suited to pupils aiming at a CSE examination (Certification of Secondary School Education).

Among the modern topics considered by the project to be included in each of the three series were probability and statistics. In the series of Book 1-5 forms for O-level students, Book 2 contained a chapter on statistics, Book T4 consisted of both a chapter on statistics and a chapter on probability, and Book T included a probability section as part of a chapter on Application of Set Theory. The statistics topics included in Book 2 and Book T4, however, are in elementary descriptive statistics.

Advanced mathematics, Book 1-4, included the material on probability and statistics in order to provide a working knowledge of some of the more important simple applications of probability theory, such as Markov chains, Poisson processes, Goodness of fit, Student's t, and Correlation and regression.

Book E, from the series Books A-H for comprehensive school students, had a chapter on elementary probability. Book F, on the other hand, had a chapter on statistics which included topics in the area of elementary descriptive statistics. Book G contained similar statistics topics to Book F, but Book G dealt with continuous data, while Book F dealt with discrete data. The background for statistics covered in Book F and G was presented in Book B.

Some of the differences between the two series, O-level and CSE books, pointed out by the project were: CSE books placed more emphasis on practical activity and investigation as well as establishing the main points by a process of question and answer; CSE books gave more time to earlier stages, divided the original content from O-level books into several parts to allow students to absorb one point before moving on to its development. These earlier chapters, upon which later chapters depend, were designed as an integral part of the book.

The content of the SMP texts differs from traditional texts because of differences in philosophical background. In constructing the series, it was the project's philosophy that: first, understanding and interest in general statements of mathematics stem from experience of a wide range of particular situations and from confidence that questions of mathematical significance can be asked and answered in many of those situations; and secondly, these experiences should arise inside as well as outside mathematics.

In the instruction of statistics in particular, the project recommended having students work with meaningful data related to themselves for motivating purposes:

> . . . Success depends to a large extent on whether the children work with data that concern and interest them.

> They seem to prefer biographical or physical data about themselves, and second best, about their friends and relations. It is compiling this that takes the time. Of course ready made data may be used. These can be obtained from encyclopaedias, daily newspapers, news weeklies . . . Children, though, seem to find these second-hand figures less interesting. It is more satisfactory if time can be allowed for the collection of the data they like best. It is a good idea to record these in a special exercise book, as the same figures can be used in several of the exercises. . . . (SMP, 1966, p. 20)

Oxford Middle School Mathematics (1971)

The project produced a four book series which aimed at providing a course in mathematics which is modern in context and reflects present day developments in helping children learn through their own activities.

The project claimed that the series was good not only for the needs of the middle schools, but was also appropriate for both upper classes in primary schools and lower classes in secondary schools. In particular, it could well provide a course to bridge the gap between the primary and secondary stages.

Each of the four books in the series divided the mathematical content into three main categories: (1) sets and numbers and number system, (2) shapes and their properties, and (3) measures, statistics and probability. The topics on statistics and probability included in the series were: collecting information and recording it in various ways—pictogram, block graph, arrow graph, reading and interpreting graphs; frequency and mode; and idea of most likely result. These appeared in Book one. Book two contained such topics as collecting data and representing it graphically, and simple ideas of sampling and probability. Book three consisted of grouped data, use of a histogram, ideas of a scatter graph, activities involving probability. Book four of the series included the idea and use of arithmetic mean, scatter graph, first ideas of correlation, probability trees.

The list of topics in statistics and probability included in the series as described above indicates a fairly large amount of content for children ages eight to thirteen. This illustrates that the project is a strong advocate of including statistics and probability in the school curriculum.

Mathematics Program in Alberta

In order to get a complete picture of mathematics instruction in the Province of Alberta, the mathematics program is reviewed at all levels—senior high, junior high and elementary. Furthermore, special attention is focused on the instruction of probability and statistics in order to see it in relation to this study.

In senior high school, according to the mathematics curriculum guide of the Department of Education (1971), the curriculum in mathematics consists of three programs, called Program A, Program B, and Program C.

Program A, the academic stream, consists of four courses—Math 10, 20, 30, and 31—which emphasize the deductive approach. Program B, the middle stream, consists of three courses—Math 13, 23, and 33—which emphasize the inductive approach, and Program C, the vocational stream, consists of two courses—Math 15 and 25—which emphasize the practical approach. It is estimated that 40-60% of the student population will take the academic stream, 25-35% the middle stream, and 15-25% the vocational stream.

Each of the three programs has a section relating to probability and statistics. For Program A, Math 30 contains a section called probability function including sample space and probability, the addition theorem, and the multiplication theorem. This section appears in Chapter 16 of the recommended text, Modern Intermediate Algebra, Revised Edition (1969). The curriculum guide recommends ten periods, out of a total of one hundred and fifty, be given to the teaching of this material.

For Program B, Math 13 contains a section called an introduction to descriptive statistics which includes: definition, significance and relevance of statistics in modern society; operations with significant digits and approximate numbers; measures of central tendency (mean, median, mode); and application. Ten periods from the total of one hundred and fifty is the recommended time allotment for teaching this section which appears in the first chapter of the recommended text, Principles of Mathematics, Book 1 (1969).

For Program C, one of the five suggested sections of Math 15 is probability and statistics with no further detail given on the topics covered. The Math 15, 25 curriculum guide (1975) indicates that teachers should feel free to modify the program to suit the needs and interests of their students, and such modification might include deletion and/or addition of some topics. This seems to imply that probability and statistics are not a requirement for students in Program C.

Mathematics in junior high school, on the other hand, is provided as a single core program with its content distributed through three years of study. The program is relatively academic in nature. Every student is expected to cover the complete program, but teachers or schools may be flexible in the degree of complexity of concept development for each individual student. Apparently the program of studies does not include any topics on probability and statistics. However, a section on probability and statistics does appear in each of the three recommended texts by the Department of Education (1976). Therefore, probability and statistics in Alberta junior high schools, if treated at all, is taught on an optional basis.

One of the three recommended text series is the Contemporary Mathematics Series. Chapter 14 of Book 3 includes a section entitled "Probability." Topics included are: outcomes, outcomes of event, mutually exclusive event, independent event, empirical probability and relative frequency.

The second recommended text is the series Mathematics Concepts and Application. Chapter 19 of the first course in the series contains a title "Statistics and Probability." Topics included in this chapter are: presentation of data, measures of central tendency (mean, median, mode), probability, sample space, estimating probabilities, and probability problems.

The third recommended text is the series Exploring Modern Mathematics. Chapter 10 of Book 3 includes a section entitled "Probability." Topics included are: what is probability; events not equally likely; compound probability, P(A or B), P(A and B).

At the elementary school level, conceptual development of mathematics is designed under three main categories: number and numeration systems, measurement and relationship, and geometry—shapes, space and location. Under the second main category—number and numeration systems—there are two topics which relate to the concept of probability and statistics. The first one is entitled "Concept of Relationship of Size, Position, Form, Quantity." Details included are: gathering data, graphical representation, and probability. The second related topic is entitled "Graphing of Simple Relationships." Details included are: pictographs, bar graphs, circle graphs, and line graphs.

In conclusion, probability and statistics have already been introduced in Alberta schools. However, the teaching time is still a very small portion of the total time allotment for mathematics, and the instruction is on a relatively optional basis. Moreover, the emphasis is generally on probability, while statistics, if treated at all, is only descriptive statistics.

RELATED RESEARCH

This study, in its nature, is a curriculum experiment. The scope of the review of previous research related to this study is primarily concentrated on the research involving curriculum experiments on statistics and probability, in particular statistics, at the level of junior high and elementary schools. Because of the contrast in content, the review of related previous research is divided into two parts: research before the sixties, and research after that time.

Research Before the Sixties

One of the oldest curriculum studies on statistics was done by Hausle (1937). The study was conducted in order to answer the question, "Can a course in statistics be given to secondary school pupils?" The question was relevant since, at that time, speaking of a course in statistics one usually thought of a graduate course in college.

The experiment was conducted at the James Monroe High School, New York City, where twenty-four boys and girls were enrolled in the course. The prerequisite for entering was the successful completion of intermediate algebra and, in order to assess the students' ability to do statistical work, a test in arithmetic and algebra was set for the beginning of the course.

The actual course instruction started out with the discussion of certain techniques for handling data. Measures of central tendency, including mean and median, were discussed and calculated, as were measures of variability. Much time was spent on graph work. Normal curve, theory of sampling and statistical inferences were also included. The course concluded with a study of correlation and regression lines.

Hausle reported that as far as possible the material used in the class was drawn from the students' own data or from data supplied by the social studies, biology, and health education courses. No report on achievement testing at the end of the course was given. However, Hausle did comment on the students' reactions to the course.

> . . . The comments of the pupils who took the course indicate that they derived a real benefit and enjoyment from it because they could see a meaningful application of their mathematics . . . (p. 29)

Hausle concluded that a course in statistics ought to become a unit in every high school wherever possible. And in such incidences mathematics teachers had to be prepared for this field.

Another early report on curriculum studies on statistics was by Drake (1941) upon the completion of his six year attempt to develop a unit of instruction based upon statistical concepts. The project was conducted at the University of Minnesota High School with ninth grade mathematics pupils.

The general objectives of the project were set up as follows (p. 16):

1. To show the usefulness of a precise numerical description of a group when measures are given for each individual in the group.
2. To teach students to arrange data so that they can be better understood and used.
3. To determine values which are representative and typical of the entire set of data.
4. To compute other values which give a better interpretation to the individual items which compose the data.
5. To show the students the limitation of such statistical procedures and possible misleading conclusions.
6. To show the need for exercising judgement and common sense in computing from approximate data.
7. To train the pupils in habits of accuracy and neatness.
8. To aid the pupils in the acquisition of concepts of statistical measurement and the requisite vocabulary.
9. To arouse an awareness of the wide spread use of statistics as a means of expression of ideas.
10. To help the pupil in his understanding of everyday problems by employing social data.

The teaching methods utilized were based on the discussion and development of concepts such as the need for arranging data; the histogram; the standard procedures for making a frequency distribution and determining the size of interval; the idea of average; the development of a formula for mean; the computation of the mean—by a regular procedure of averaging and also by short cut procedures (assuming the midpoint of the interval as the score obtained by all the students in that interval, and by using a midpoint of some intervals as the assumed mean, then the adjustment was made later); the median; the use of the mean and the median; and percentile, quartile, decile. Drake also had students work with their own testing scores during the instruction.

Drake assessed the unit and made recommendations as follows:

> . . . The pupil interest and participation were high during the presentation of this unit. The student teachers who were observing remarked that they themselves had a much better understanding of statistics as a result of their experience. . . . Possible improvements lie in the introduction of more social data and less emphasis on test scores. . . . (p. 22)

Similar studies were conducted later, by O'Toole (1952) about statistics in secondary school curriculum, by Beberman (1953) about the teaching of statistics in secondary school mathematics, and by Krulik (1959) on experiences with some different topics, including statistics, for slow learners. These studies before the sixties primarily involved statistics—in particular, descriptive statistics—although the study by Hausle (1937) did include topics involving inferential statistics such as the theory of sampling and statistical inference. But according to his report, these were offered only to students who had completed an advanced course in algebra. Furthermore, although it was suggested that real data be employed in the investigation, the emphasis was on mathematical elements, formulas, and computations. For instance, there was a heavy emphasis on various procedures in computing the mean as described by Drake (1941).

Research in the Sixties and Later

Starting with the sixties, studies involving probability and statistics at the secondary and elementary school levels came into fashion. Most of these studies show a lot of emphasis on probability concepts, although some do refer to both probability and statistics.

Leake (1962) conducted a status study on the ability of seventh, eighth and ninth grade students to understand the concepts of probability. The study showed that the children had the ability to understand a considerable number of probability concepts. This seemed to imply that probability and statistics can be included in the school curriculum for young children. However, he suggested that in order to assess the feasibility of such inclusions, curriculum experiments be done first on the various aspects of teaching probability and statistics at that level.

The following is a more detailed review of some of the curriculum studies on probability and statistics conducted during the sixties and later at the level of junior high school or below.

The first pertinent curriculum experiment closely related to this study was conducted by Shulte (1970). The main purpose of the study was to investigate the results of the introduction of a unit on probability and statistics into the ninth grade general mathematics course.

During the summer of 1966, the researcher developed a 199 page instruction unit, Mathematics of Uncertainty, and a 97 page teachers' guide for the unit. The prepared unit consisted of nine chapters:

1. Introduction

2. Probability—a Beginning (simple events, repeated events, independent events, mutually exclusive events, empirical probabilities, conditional probabilities, tree diagrams, odds)

3. Fancy Counting (permutations, combinations, the Pascal triangle)

4. Probability Revisited (a look at probability using permutations and combinations)

5. Getting Organized (tables and statistical graphs)

6. Describing Data (measures of central tendency—mean, median, mode; measures of position—quartiles, percentiles; measures of spread—range, trimmed range, average deviation, standard deviation; mean and standard deviation for group data; mean and standard deviation of a probability distribution)

7. Sampling, Prediction, and Quality Control

8. How are Things Related? (regression and correlation)

9. Suggested Projects (a list of forty possible student projects).

During the 1966-67 school year the unit was taught to eighteen classes of ninth grade general mathematics students in five school districts of Oakland County, Michigan. Thirteen teachers volunteered to teach the unit. Seventeen classes of ninth grade general mathematics students in the same five districts, using their regular materials, were used as a comparison group. The comparison classes were taught by eleven volunteer teachers. The researcher held four in-service training sessions of one and a half hour each, prior to and during the experiment, to provide the teachers of experimenting classes with a common base of knowledge about probability and statistics and the particular views of teaching.

The intended time for teaching the unit was about eight or nine weeks of the school year. The actual teaching time ranged from five to fifteen weeks depending upon the individual district. Moreover, the decision to stress or omit any materials was left to the individual teachers.

The data were collected by means of pretests and posttests. Standardized tests were mostly used in this study, except two of those tests which evaluated the teachers' attitude and the students' achievement. These were developed by the researcher.

Five main hypotheses with appropriate subhypotheses were statistically tested. Some conclusions were:

1. The study of probability and statistics was not beneficial in improving computational skills, except for multiplication. Moreover, the computational skill for division had significantly declined.

2. Probability and statistics contributed to the improvement of pupil performance in four selected mathematical areas: (a) statistical graphs and tables, (b) mathematical symbolism and formulas, (c) solution of simple equations, and (d) mathematical achievement.

3. Ninth grade general mathematics students learned concepts and principles of probability and statistics.

4. The study of probability and statistics caused student attitude toward mathematics to decline.

5. Teacher attitudes did not appear to change.

One could look at the study by Shulte as a comprehensive one, since it involved a large number of students and teachers, and also a substantial unit of content. However, the instructional approach and the teaching strategies during the experiment were relatively uncontrolled. The decline of the students' attitude and computational skill in division may have been due to this uncontrolled major variable.

A second study of curriculum experiment, also closely related to this study, was conducted by Smith (1966). The main purpose of Smith's study was twofold: (1) to develop materials in probability and statistics for seventh graders, and (2) to evaluate the effectiveness of those materials.

The developed materials consisted of a teacher's and student's manual which covered eleven elementary topics. The objectives for the eleven topics were formulated under the categorization of knowledge, understanding and abilities. An objective test consisting of fifty items was used as a pre- and posttest measure. The eleven topics were:

1. Equally likely events
2. Events that are not equally likely
3. Mutually exclusive events
4. Independent events
5. Models
6. Sampling
7. Pascal's triangle
8. Continuous and discrete data
9. Histographs and frequency polygons
10. Central tendency, including mean, mode, median

11. Variation, including range and average deviation.

The materials were taught to four classes, totalling ninety-seven students, for seventeen days. The control group included two seventh grade classes totalling fifty-three students. Some conclusions from the study were:

1. High and middle ability students learned significantly more than those designated as low ability students.

2. Low ability students did learn a significant amount.

3. All but three of the eleven topics—independent events, sampling, and measures of variation including average deviation—were appropriate for those low ability students. However these three topics did seem to be appropriate for high ability students.

Smith's study seemed to demonstrate the feasibility of seventh grade students learning certain topics in probability and statistics. However, the statistical concepts taught were mostly in the area of descriptive statistics, and the unit is relatively fragmented. Moreover, seventeen days of instruction to cover eleven topics does not seem to be enough time to guarantee the students' good performance, especially those students of low ability.

A third curriculum experiment was conducted by Wilkinson and Nelson (1966). The purpose of the study was to test the feasibility of students learning certain concepts of probability and statistics. The sample consisted of 22 sixth grade students from the Laboratory School, State College of Iowa. The study was conducted for three weeks of 45 minute per day periods. The class was taught by one of the researchers, while the other acted as an observer-recorder. No formal testing was done at the end; only subjective information was reported.

The selected concepts which they intended the students to experience were: randomness, probability, odds, bias, variation, law of large numbers, level of confidence, sampling, mean, median, mode, distributions, equally and unequally likely events. The skills which they intended the students to develop were: recording and interpreting data, making and testing hypotheses, and observing and differentiating between relevant and irrelevant data.

The conclusions, subjectively drawn by the researchers, were that the experiment was a qualified success, that probability experiences were worthwhile, and that it is important to expose students to events which have a degree of uncertainty. Wilkinson and Nelson further conjectured that activities in this grade should enable students to compare their individual guesses with what they find taking place as they carry out an experiment, and that a series of experiments involving possibility concepts would help them assess and interpret uncertainties.

The results of this study seem to be useful, but the generalizations may not be convincing because of the subjective nature of evidence cited in the conclusion.

A fourth curriculum experiment was conducted by Shepler (1969). The twofold purpose of the study was to construct an instructional program on probability and statistics and then to determine the success of teaching the unit to sixth grade students. The content outline was hierarchically analyzed and consisted of nine lessons based on the following topics:

1. The use of experiments by children to provide an empirical approach to the subject.

2. Counting paradigms (trees, table, lattices).

3. Formal counting procedures (sum rule, product rule, permutations, and combinations).

4. Relative frequency of an event to approximate the probability of that event.

5. Computation of probabilities in a finite sample space.

The study was conducted in the Waunakee Elementary School, Waunakee, Wisconsin in March and April 1969, using 25 average and high I.Q. students. The teaching staff consisted of one teacher and the researcher. Since mastery learning procedure was employed as the major goal of the experiment, three more teachers and a secretary were used as auxiliary staff during the course of instruction.

The data were gathered at the end of the experiment by employing a criterion referenced test to determine the students' level of mastery with respect to a set of behavioral objectives.

Shepler's study seemed to be very successful because about 98% or more of the students achieved about 90% or more of the test items set as the criterion by the researcher. However, the study does not seem to represent a typical teaching and learning situation in a general classroom because five people were employed to teach and help a class of twenty-five high ability students. The distinct result of this study seems to illustrate the effect of mastery learning procedure rather than the success in introducing probability and statistics to sixth grade students.

A similar study to Shapler's was conducted by Aggrawal (1971) in the city of Calgary, Alberta. The main purpose of the study was to test the feasibility of teaching probability to grade five students. Two groups, each consisting of fifteen students, participated in the study for ten instructional days. The instructional unit employed was primarily based on Activities in Mathematics, First Course on Probability, a Scott-Foresman publication. In addition, a film on probability was shown to the students as a part of instruction. The analysis, based on the results of pre- and posttests, was made and it was concluded that it is feasible to teach probability to grade five students.

Curriculum experiments concerned only with statistics were conducted by Grass (1965) and Girard (1967). Grass (1965) taught a fourth and fifth grade class a unit including the concepts of central tendency—mean, mode, and median. The students were helped to devise and evaluate their own statistical study to determine relationships between the extrasensory perception of boys and girls. Hypotheses were made and tested using mean, mode, and median. Students' interest and enthusiasm were reported to be very high for the unit. However, no objective basis for conclusions was made.

Girard (1967) conducted a curriculum experiment involving the development of a unit on descriptive statistics and graphing with a class of elementary students. The main purpose of the instruction was to develop children's ability to critically interpret statistics and graphs. Students were reported to be enthusiastic about the topic and willing to carry out special projects on their own. However, besides the description of the proceedings, no empirical evidence was gathered and reported.

In summarizing this review of previous related research, one sees that the studies before the sixties put emphasis on descriptive statistics and the manipulation of the mathematical elements of statistics. In the sixties and later, the studies seemed to put more emphasis on probability concepts, while statistics became secondary content.

INSTRUCTIONAL THEORIES

Introduction

The instructional unit in this study is approached in the intuitive fashion; that is, the students are expected to learn statistical concepts without formal proof. Almost all educational psychologists recognize the importance of intuitive learning to some degree, and make use of this aspect of learning in their theories. Skemp (1971) includes intuitive intelligence as a contrast to reflective intelligence in his theory. Dienes (1964) strongly advocates intuitive learning. Ausubel (1968) also recognizes intuitive learning, although to a somewhat less degree. In his words ". . . it is preferable to restrict the intuitive-oriented content of the elementary school curriculum to materials for which the child exhibits adequate developmental readiness—even if he can intuitively learn more difficult, ingeniously presented material beyond his intrinsic level of readiness. . . . (p. 213)." Bruner (1960), Dienes (1964), Arnsdorf (1961), Brownell (1960), and Davis (1958) claim that by using an intuitive approach it is possible to successfully teach young children many ideas in science and mathematics which were previously thought much too difficult. Because Bruner is the prime proponent of this claim, the first part of the review of instructional theories is devoted to his intuitive learning theory.

One main purpose of this study is to develop an instructional unit in inferential statistics. The construction of such a unit involves the preparation of materials which will guide the students into meaningful learning. The presentation of these materials to the students is supposed to affect the students' anchoring ideas about statistical concepts. The second part of the review of instructional theories, therefore, deals with Ausubel's meaningful learning theory.

Meaningful learning, according to Ausubel, can occur only when the learner has some relevant anchoring ideas to relate the new material to. Because the third purpose of this study involves the investigation of the students' anchoring ideas about statistical concepts, a review of the nature of anchoring ideas is included in the last part of the review of Ausubel's meaningful learning theory.

Intuitive Learning

Intuition, according to Bruner (1974) in his *Learning of Mathematics: A Process Orientation*, implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft. It is the intuitive mode that yields hypotheses quickly, that produces interesting combinations of ideas before their worth is known, that precedes proofs, and that furnishes proposals which the techniques of analysis and proof are designed to test and check. It is founded on a kind of combinational playfulness. It is a form of activity which depends upon confidence in the worthwhileness of the process of mathematical activity rather than upon the importance of right answers at all times.

From a psychological point of view, the child's mental development involves the construction of a model of the world in his head, and an internalized set of structures for representing the surrounding world. These structures are organized in terms of perfectly definite rules of their own. And in the course of mental development the structures change and the rules governing them also change in a certain systematic way.

Intuition, when applied to mathematics, involves the embodiment or concretization of ideas, not yet stated, in the form of some sort of operation or example. The child grasps a model of a certain approached mathematical structure which is implicitly governed by all sorts of seemingly subtle mathematical rules, some of them newly acquired and some original.

Some features of intuitive thinking that seem of particular importance to various intellectual disciplines are, according to Bruner (1973), activation, confidence, visualization, nonverbal ability, the informal structuring of a task, and the partial use of available information.

Activation is the first requirement of any problem solving sequence. It refers to getting started, getting the behavior out where it can be corrected, getting the learner committed to some track, allowing the learner to make an external summary of his internal thought process. It is in essence an imprecise opening move, a sensing of a possible or estimated way of getting started.

Although before making a start on a task one must have some degree of self-confidence, the act of starting itself increases one's confidence in the ability to carry the task through. The confidence of having made a start, along with a sense of the problem corrigibility, helps the learner to carry on the task.

Visualization refers to the perception which is the limit of most intuitive heuristics. When one says that he sees something, often he means that he senses it in a visualized or sensory embodiment. This kind of embodiment permits directness of grasp and immediateness of conclusion.

There is also an ability characteristic of intuitive thinking which is nonverbal in nature. In such a case a person is in the state of not being able to give verbal justification for why he is proceeding as he is or why he has made a particular discrimination. The learner's behavior is not fully under control in the sense of being translatable into the language necessary for summary, transformation, and criticism.

Intuition is a short cut based on an informal and often inexpressible structuring of a task. The so-called structure may be nothing more than a sense of connection between means and ends or some notion, difficult to clarify, of belongingness.

The partial use of available information involves the reducing of the range of things to which one attends. One may use a heuristic involving visualization or some other shorthand way of summarizing the connections inside a set of givens. This act requires not only a certain amount of confidence, but also a kind of implicit rule for ignoring certain information.

Regarding the translation of intuitive ideas into mathematics, Bruner maintains that anything that can be said in mathematical form can also be said in ordinary language, though it may take a long time to say it and there will always be the danger of imprecision of expression. According to Bruner, three problems are involved in the translation: the problem of structure, the problem of sequence, and the problem of embodiment.

When one tries to get a child to understand a concept, the first and most important step is that one must understand that concept well himself. To understand well means to sense the simpler structure that underlies a range of instances. It is obviously true in mathematics. Once one senses the structure one has to find a common language which will transmit that structure to the learner without much loss of precision.

The problem of sequence involves discovering a way to get the child to progress from his present concept to a more subtle grasp of the matter. Naturally, in order for a child to understand some concepts he needs to understand some other concepts first. For example, a child needs to understand the concept of serial ordering before he can be introduced to and understand the concept of transitivity.

The problem of embodiment involves discovering a way to embody illustratively the middle possibility of something that does not quite exist as a clear and observable datum. The problem is to translate a concept into a simpler homologue, an invisible object whose existence depends upon indirect information.

Bruner has summed up the importance of intuitive learning in education. In his words:

> . . . learning and teaching must start from some intuitive level. This may be true not only of young children entering the educational establishment for the first time, but of anybody approaching a new body of knowledge or skill for the first time. Only a romantic pedagogue would say that the main object of schooling is to preserve the child's intuitive gift. Only a foolish one would say that the principle object is to get him beyond all access to intuition, to make a precise analytic machine of him. Obviously, the aim of a balanced schooling is to enable the child to proceed intuitively when necessary and to analyze when appropriate. . . . (1973, p. 83)

Meaningful Learning

Meaningful learning, according to Ausubel, involves the acquisition of new meanings; new meanings, on the other hand, are the products of the meaningful learning. This means that the emergence of new meanings in the learner reflects the completion of the meaningful learning process.

The meaningful learning paradigm involves two important phenomena: cognitive structure, and the nature of material to be learned. The first one, cognitive structure, refers to the learner's present knowledge of a given discipline, consisting of the facts, concepts, propositions, theories, and raw perceptual data that the learner has available to him at any point in time. The quantity, clarity, and organization of the learner's present knowledge or cognitive structure is the most important factor which influences his learning.

The second important phenomenon is the nature of material to be learned. Materials vary in the extent to which they can be related in some sensible or understandable way to an adequate cognitive structure. In other words, materials differ in respect to the degree of relationship which they can exhibit to ideas that lie within the realm of human learning capability. This means there exists some kind of material relatability to the learner's cognitive structure which determines the type of learning. The kind of material relatability that leads to meaningful learning must possess two particular qualities—substantiveness and nonarbitrariness.

The quality of substantiveness means that the relationship is not altered if a different, but equivalent form of wording is used. For example, several equivalent definitions of an equilateral triangle maintain an unchanged relationship to the learner's idea of a general triangle presumably already known to him.

The second quality is that the relationship between the new material to be learned and relevant ideas in cognitive structure be nonarbitrary, or nonrandom. As in the above example, the relationship between "an equilateral triangle" and "a triangle" is certainly nonarbitrary because it is the relationship of specific instance to general case.

Material possessing the two specific properties of relatability—nonarbitrariness and substantiveness—is in the condition called "logical meaningfulness." Evidently the material will vary in logical meaningfulness to the degree that it possesses these two underlying qualities.

The fact that the material is logically meaningful means only that it could be related to ideas that lie within the realm of human learning capability. In order for the material to be understandable by a particular individual, that individual himself must possess the necessary relevant ideas in his cognitive structure. If a particular learner does possess relevant ideas in his cognitive structure to which the new material can be related substantively and nonarbitrarily, then that material is said to be "potentially meaningful" to him.

The material is now in the condition of potential meaningfulness. In other words, it can be related nonarbitrarily and substantively to some hypothetical cognitive structure of a particular learner who does possess the relevant ideas. All that the learner needs is his "intent" to do so. If the learner has a "meaningful learning set," which is the intent to relate the material nonarbitrarily and substantively to relevant ideas in his cognitive structure, then meaningful learning takes place.

Meaningful learning, in summary, requires that these three conditions hold: (a) the material must be relatable to some hypothetical cognitive structure—nonarbitrarily and substantively, (b) the learner must possess relevant ideas in his cognitive structure to relate the material to, and (c) the learner must possess the intent to relate the material to his cognitive structure nonarbitrarily and substantively.

Meaning now occurs as the completion of a meaningful learning process. It is the differentiated cognitive content, or content of awareness, which emerges when potentially meaningful material is incorporated into cognitive structure in a nonarbitrary and substantive fashion.

According to Ausubel, there are three kinds of meaningful learning—representational learning, propositional learning, and concept learning. Representational learning refers to the learning of the meaning of individual symbols, such as spoken or written words representing objects. Propositional learning refers to the learning of the meanings of sentences or syntax. And concept learning refers to the learning of the meanings of a class of objects that are grouped together because of their common characteristics or criterial attributes. There are two types of concept learning, called concept formation and concept assimilation. Concept formation is the typical learning of young children who inductively discover the criterial attributes of a class of stimuli by themselves. Concept assimilation, on the other hand, is the typical learning of school children, in which the criterial attributes of a concept are generally presented to them by some kind of definition. In relation to the instructional unit in this study, the concept learning is more of the concept assimilation type than concept formation because students at the grade nine level have a good command of the language and have relevant experiences to which the new concepts can be related.

Anchoring Ideas

According to Ausubel, there are three principal variables influencing meaningful reception learning. The first important variable affecting the incorporability of new meaningful material is the availability in cognitive structure of relevant anchoring ideas or subsuming concepts at an appropriate level of inclusiveness to provide optimal anchorage. A second important factor affecting the learning retention of a potentially meaningful learning task is the extent to which it is discriminable from the established conceptual systems that subsume it. And the last one is that the learning and longevity in memory of new meaningful material are functions of the stability and clarity of its anchoring ideas.

Each of the three principal variables described above involves the nature of anchoring ideas as they affect learning. In relation to those three variables, availability, discriminability, and stability of anchoring ideas, Ausubel further illustrates the situation. If specifically relevant ideas are not available in cognitive structure when new potentially meaningful material is presented to a learner, what happens? If some existing, though not entirely or specifically relevant set of ideas cannot be utilized for assimilative purposes, the only alternative is rote learning. More typically, however, less specifically relevant ideas are pressed into service. The outcome is either a form of combinational assimilation, or less relevant correlative subsumption. As a result, in either case, less efficient anchorage of the new material to cognitive structure occurs, bringing about ambiguous or relatively unstable meanings with little longevity. These ambiguous and unstable ideas not only provide inadequate relatability and weak anchorage for potentially new materials, but also cannot be discriminated from them.

Sometimes appropriately or specifically relevant subsumers are available, but their relevance is not recognized. In such cases the same outcome as above may result. In either case, for meaningful verbal learning situations, it is preferable to introduce suitable "organizers," or introductory materials at a high level of generality and inclusiveness in advance of learning material. The introductory material serves an assimilative role, replacing reliance upon the spontaneous availability or use of less appropriate anchoring ideas in cognitive structure.

SUMMARY

Several influential reports recommend the inclusion of probability and statistics in school curriculum from elementary to high school. The mathematics program in Alberta also indicates the movement toward that trend by including some topics of probability and statistics scattered through the school curriculum. However, the instruction of the subject at junior high school level or lower is only an option, and constitutes a very small portion of teaching time.

The amount of empirical evidence available for objectively judging the feasibility of teaching probability and statistics at the junior high school level or lower is relatively small. However, curriculum experiments incorporated with some learning theories as a basis for instructional approach do suggest that teaching certain concepts of probability and statistics to young students at the level of junior high school or lower may be feasible.

CHAPTER 3

CONSTRUCTION OF THE UNIT

One main purpose of this study is to construct a unit in inferential statistics for grade nine students. This chapter describes the detail of that construction. The beginning section of the chapter is about the assumptions concerning the constructibility of such a unit, followed by the description of the general plan employed as a guideline in the unit construction. The three main aspects of the general procedure are: content analysis, instructional analysis, and pilot study.

ASSUMPTIONS

As a starting point for the construction of a unit in inferential statistics, the assumptions about the unit constructibility in relation to the approach, content, learner, and major prerequisite course are stated as follows:

1. Approach: Bruner (1960), in his Process of Education, makes the hypothesis that:

> Any subject can be taught effectively in some intellectually honest form to any child at any stage of development . . . (p. 33).

One way to achieve the aim suggested by the above hypothesis, according to Bruner, is to make use of the learner's intuition. This brings about the use of an intuitive approach in teaching a subject. The intuitive approach is still a general approach which needs clarification with respect to the context, in this case the development of a unit in inferential statistics. It is assumed that an appropriate intuitive approach can serve as the instructional means to enable the majority of grade nine students to adequately learn the concepts of inferential statistics, and that it is possible to build such an approach into the unit during its development.

2. Content: Bruner (1974), in his "Learning of Mathematics: A Process Orientation," states that:

> When we try to get a child to understand a concept, leaving aside now the question of whether he can "say" it, the first and most important problem obviously, is that we as expositors understand it ourselves . . . to understand something is to sense the simple structure that underlies a range of instances, and this is notably true in mathematics. (p. 172)

The second assumption about the unit development is related, in a sense, to the statement by Bruner in that it is assumed that the content of inferential statistics can be reduced to an appropriate simple structure such that it can be adequately learned by grade nine students.

3. Student Background: Sawyer (1973), in his book "Mathematician's Delight," in answer to questions such as, "Why should such fear of mathematics be felt?" "Does it lie in the nature of the subject itself?" "Are great mathematicians essentially different from other people?" remarks that:

> . . . quite certainly the cause does not lie in the nature of the subject itself. The most convincing proof of this is the fact that people in their everyday occupations—when they are making something—do, as a matter of fact, reason along lines which are essentially the same as those used in mathematics: but are unconscious of this fact, and would be appalled if anyone suggested that they should take a course in mathematics. (pp. 7-8)

The third assumption in relation to the unit development concerns the justification for believing that grade nine students have adequate background in statistical reasoning appropriate for the study of a unit in inferential statistics. In several instances of everyday situations students unconsciously know how to use the important basic concepts of inferential statistical reasoning. For instance, if the students are asked, "What do African people look like?", normally what they have seen of some African people will help them point out some characteristics as an answer to the question. This means the students unconsciously use the very important basic inferential statistical way of reasoning—generalizing from a particular to a general or from the seen to the unseen, which is equivalent to reasoning from a fraction of a whole (sample) to the whole (population).

It is assumed that the above mentioned inferential statistical reasoning serves as the students' relevant anchoring idea, making it possible for them to understand the materials to be developed in the unit. And the relevance of such background will be incorporated into the instruction of the unit.

4. A unit in inferential statistics without a prior course in probability: It is assumed that inferential statistics with its special mathematical features can be adequately learned by students who do not have a prior formal course in probability. However, some intuitive concepts of probability may need clarifying, but those concepts can be introduced to the students during the process of the unit instruction. As evidence to support this justification, a lot of people who know statistics quite well often admit that they do not

know very much about probability, which actually refers to the formal theory of probability.

The National Advisory Committee on Mathematics Education (1975) makes a statement concerning the present shortage of good statistics texts in the schools. They feel the shortage is due to the fact that generally statistics is treated as merely the application of the probability theories. This committee's statement agrees, in a sense, with the assumption that statistics can be independently taught:

> In spite of the rather widespread belief that statistics is a branch of applied mathematics, statistics has important extra mathematical features to be presented along with the mathematical method. In contrast to the case of probability proper, interesting and authoritative teaching materials in statistics at the precollege level are hard to come by. (p. 46)

In summary, one can make the following assumptions about the constructibility of a unit in inferential statistics: The intuitive approach employed in the unit can be appropriately devised so that the majority of grade nine students can adequately learn the statistical concepts to be taught. The content of inferential statistics can be reduced to an appropriate simple structure to be adequately learned by grade nine students. Grade nine students are assumed to have relevant background, in particular familiarity with important basic inferential statistical reasoning by which they can further study the unit in inferential statistics. And a unit in inferential statistics can be constructed and taught to grade nine students without heavily relying on the theories of probability.

GENERAL PLAN FOR THE UNIT DEVELOPMENT

In a mathematics program of instruction, according to Gagné (1974), there are two broad categories of factors which potentially influence mathematics learning. The first of these is called the "instructional variable"; and the second category involves the content which is called "topical order." Romberg and Devault (1967) have presented a developmental model of the instructional program. The first stage of their model is the analysis stage which involves an analysis of the mathematical content, followed by an analysis of how to communicate the content to the students. It is apparent then, that these two sources share the opinion that instruction and content are the two major variables influencing mathematics learning.

These two major variables were used at the beginning of the unit development, adapting the analysis stage of Romberg and Devault. The pilot study was conducted in order to judge the appropriateness of the unit before going on to the final form. Figure 3.1 illustrates the flow chart of the general plan which was used as the main guideline for the unit development.

CONTENT ANALYSIS

The content analysis starts with overviewing a simple structure of inferential statistics in which one can see the important components and the relationships among them. The overview of the structure, incorporated with the content recommended by some influential reports, brings about the outline of the content for the unit as described in the second section. The third section deals with

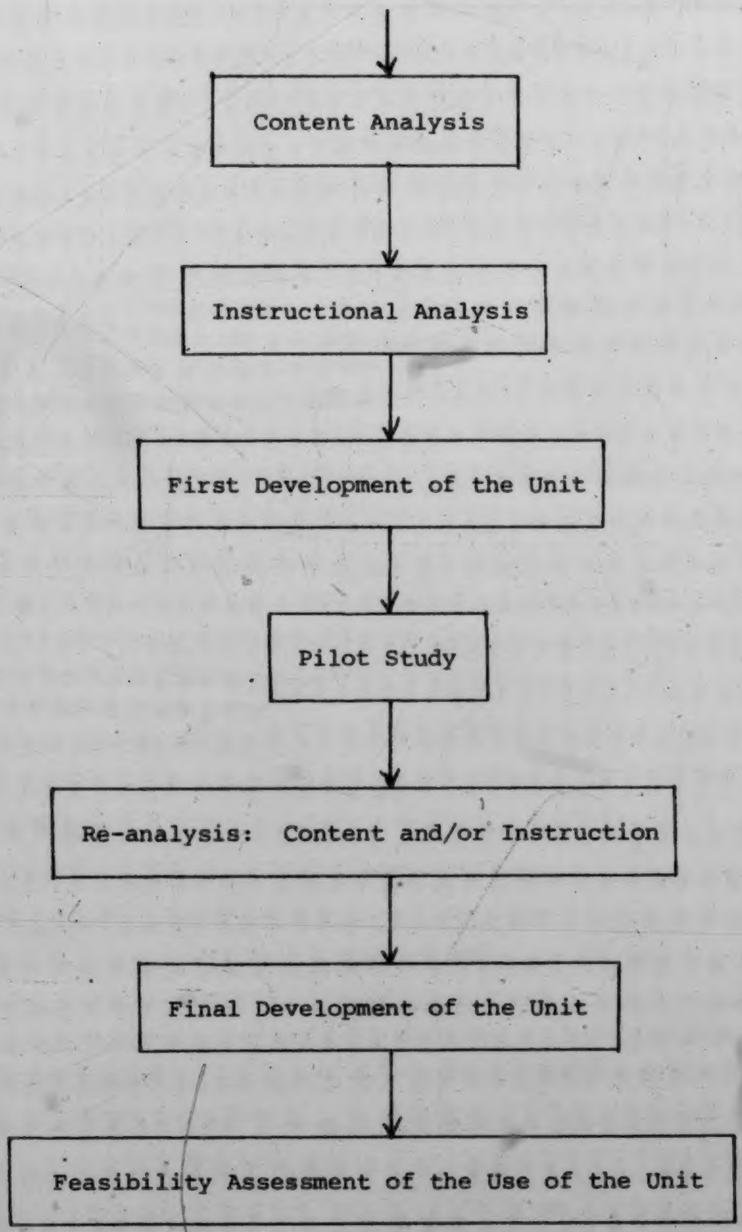

Content Analysis
Instructional Analysis
First Development of the Unit
Pilot Study
Re-analysis: Content and/or Instruction
Final Development of the Unit
Feasibility Assessment of the Use of the Unit

Figure 3.1

General Plan for the Unit Development

the prerequisites, while the fourth section specifies the objectives of the unit in terms of student achievement.

Overview of a Structure of Inferential Statistics

The simplest structure of inferential statistics can be viewed as consisting of three basic components:

1. A set of data which is referred to as "a population."
2. A subset of a population which is referred to as "a sample."
3. An attempt to get the knowledge of a population by the information from its samples.

The first component of the simple structure is a population. A set of data, either finite or infinite, that one wishes to study or about which one wishes to make inferences is considered a population in the general sense. A population possesses certain characteristics that describe its various aspects, such as, a distribution, central tendency, and variability. These characteristics of a population are called "parameters." Therefore a parameter describes a population.

Similarly, the second component is a sample which represents a part or a subset of a population. A sample also possesses certain characteristics equivalent to those of a population, such as, mean, median, mode, and variance. In contrast to parameters of a population, these characteristics of a sample are called "statistics." A statistic describes a sample in the way a parameter describes a population. Therefore Statistics (a subject), which only involves the manipulation of characteristics from a set of data, either a sample or a population, in order to describe that set of data, is called "descriptive statistics."

The third component is an attempt to make inferences about a population by using information from a sample taken out of that population. There are two very important basic ideas under this component; they are estimation and hypothesis testing. The idea of estimation is to produce a statistic from a sample which is regarded as an approximation of an equivalent parameter of the population from which the sample was taken, as in the case of a sample mean as the estimate of a population mean. Hypothesis testing is the idea of judging the status of a parameter statement, that is, a statement about a population's characteristics, whether it is true or false with regard to a certain criterion applied to information from a sample. As in the case of testing the simple hypothesis, or statement, that a population mean equals a certain constant, a sample mean is used along with some statistical procedures to judge the original statement.

From the simplest structure consisting of the three basic components, one can move to a more complicated structure by adding various statistical concepts to each of the three components. Following are some important basic concepts of inferential statistics which were added to the structure.

The Concept of Random Sampling: This concept can be viewed as belonging under the third component of the structure—an attempt to get the knowledge of a population by its samples. It is a procedure for selecting an unbiased sample from a population and may be either a simple random sampling, such as sampling with or without replacement, or a complex random sampling, such as stratified sampling, cluster sampling, and two stage sampling. This concept is very important to inferential statistics. It involves samples, estimates calculated from them, and the ideas of characteristics of the population sampled.

The Concept of a Sampling Distribution: This concept is basic to the entire branch of inferential statistics. A statistic or an estimated value calculated on a sample is said to possess a certain sampling distribution. As an example, a sampling distribution of a mean of a sample taken from a certain population is said to possess a normal distribution. The concept of sampling distribution helps us to know the behavior of a statistic, which is a mathematical fact, in order to assess the representativeness to be expected from random samples thus leading to the knowledge of a population.

The Concept of a Normal Distribution: The concept of a normal distribution is very important in inferential statistics. Many statistical techniques rest on the assumption that the frequency distribution of scores on a variable in a population is adequately described as a normal distribution, and many other important theoretical distributions, such as t-distribution, chi square distribution, and F-distribution, are based on the normal distribution theory. A normal distribution is attributed to both empirical fact and mathematical fact. As an example of empirical facts, the biological measures such as height, weight, and intelligence of a large number of people are found to possess a normal distribution; and so is a distribution of errors in measuring an object. For mathematical fact, the concept of a normal distribution itself is a mathematical invention which is determined by two parameters, mean and standard deviation. The concept of a normal distribution plays an important role in a well known mathematical theorem—the central limit theorem. The theorem essentially states that the distribution of the means of random samples taken from a population with some form of distribution will approach a normal distribution as the sample size increases.

Content Outline

In selecting the statistical content to be included in the unit, three bases for consideration were used—psychological, social, and mathematical.

The psychological basis for justification of course content in this study relies on Bruner's point of view regarding the structure of a subject matter and regarding cognitive development. Bruner (1960) emphasizes the importance of a structure: (1) Understanding a structure—the fundamentals—makes a subject more comprehensible, (2) It relates to human memory in that if detailed material is placed into a structure pattern it is conserved in long term memory, and (3) Understanding the structure of a subject matter is the main road leading to adequate transfer of training to application.

In Bruner's theory of cognitive development, he suggests that a child internalizes learning materials via three modes of content representation: a set of actions—enactive, a set of pictures—iconic, and a set of symbols—symbolic. Bruner further suggests that any subject can be taught to any child in an intellectually honest form, by which he means the content structure to be taught can be put into an appropriately simple structure and then presented through appropriate learning representations as mentioned.

Bruner's point of view, therefore, concerns the developmental process in learning and the nature of the content to be learned. The consideration in selecting the statistical content to be included in the unit is concerned with the question of what statistical content, in particular what aspects of the structure of basic inferential statistics, is considered to be worth including in a course for grade nine students according to Bruner's point of view. Furthermore, what is the most advanced statistical topic to be included in the structure such that the content consisting of that advanced topic, along with the subordinate topics, can be appropriately modified and presented through appropriate learning representations for grade nine students.

Social justification is based on the reports of various influential studies, such as those of the Cambridge Conference, the College Entrance Examination Board and others, that recommend the inclusion of probability and statistics in the school curriculum on the basis of its importance and usefulness for various occupations and everyday life in modern society. The College Entrance Examination Board Commission on Mathematics (1959) recommended the inclusion of a course in probability and statistics in the secondary school curriculum and emphasized its importance as follows:

> So great is the current scientific and industrial importance of probability and statistical inference

> that the Commission does not believe valid objections based on theoretical considerations can be offered to its inclusion in the curriculum. . . . Not only is there a great demand in a wide variety of occupations . . . but this material, more than most secondary school mathematics, is closely related to problems of daily living (p. 32).

Furthermore, various influential reports recommended content outines which proposed the inclusion of several statistical topics. These statistical topics are taken into consideration in selecting the statistical content to be included in the unit.

Mathematical justification is based on the mathematical features of statistics illustrated by the use of mathematical procedures. As pointed out by the National Advisory Committee on Mathematics Education (1975), ". . . statistics has important extra mathematical features to be presented along with the mathematical method" (p. 46); statistics is a kind of mathematics and any statistical concepts can be viewed as mathematical concepts which form the structure of statistics. Therefore, in considering the statistical content to be included in the unit with respect to a mathematical basis for justification, emphasis was placed on the selection of basic statistical concepts to form a statistical structure, and the use of mathematical procedures to illustrate those various statistical concepts.

Two reports, among several that advocate the inclusion of statistics in school curricula, have presented course outlines. The first report, the Cambridge Conference on School Mathematics (1963), has proposed mathematics curricula for grades 7-12, presenting two topical outlines. Both outlines have included probability and statistics at the junior high school level, the most advanced topics included being statistical estimation and hypothesis testing. The recent report by the Committee on the Undergraduate Program in Mathematics—Introductory Statistics Without Calculus (1972), has also proposed a course outline which includes such topics as statistical description, normal distribution, central limit theorem, point estimation, and confidence interval estimation.

Along with the psychological, social and mathematical considerations in selecting content, the choice of content for the unit in this study follows closely that content recommended by these two influential reports; statistical estimation being selected as the most advanced topic in the unit. In order to fit the unit into the structure of inferential statistics as described in the first section of the content analysis, some fundamental statistical concepts were added to each of the three components. Figure 3.2 illustrates the development of the content outline with respect to a defined structure of inferential statistics as employed in this study.

The first component consists of the concept of a population along with the description of some of its characteristics or parameters. The second component consists of the concept of a sample and also the description of some of its characteristics or statistics, such as, a frequency distribution, central tendency, variability, and score transformation. The third component, an attempt to link the first two components to get the knowledge of a population, consists of the concepts of simple random sampling, a sampling distribution, and statistical estimation.

The concepts of frequency distribution, measures of central

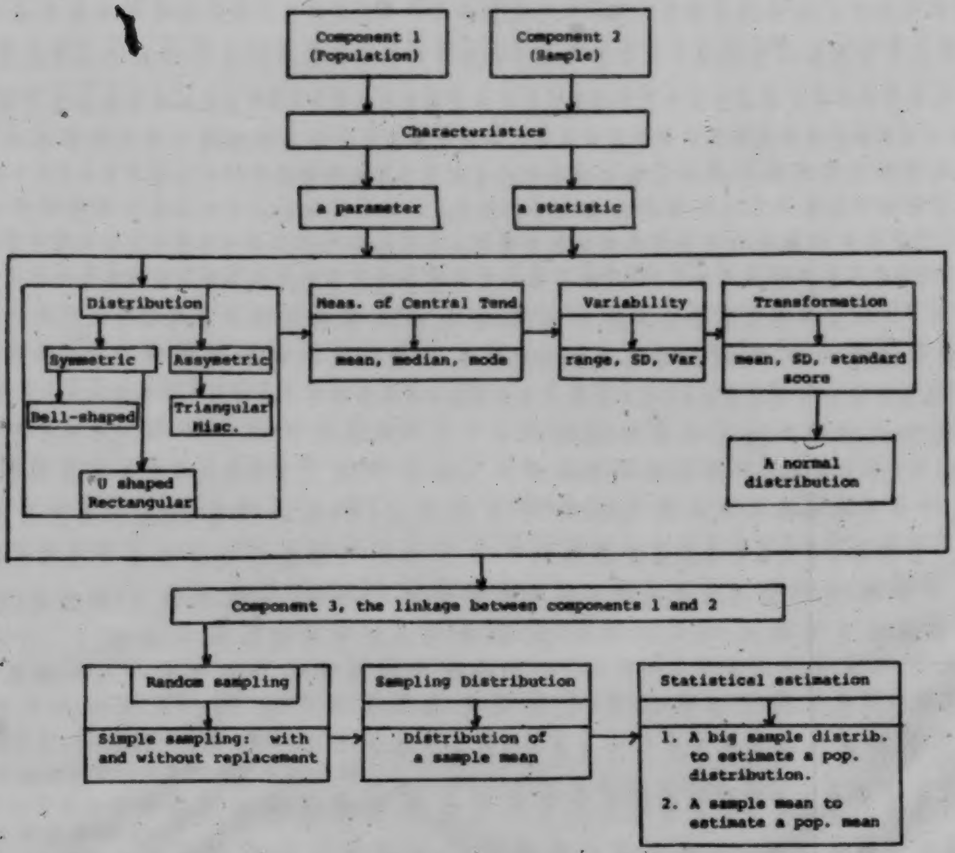

Component 1
(Population)
Component 2
(Sample)
Characteristics
a parameter
a statistic
Distribution
Symmetric
Assymetric
Bell-shaped
Triangular
Misc.
U shaped
Rectangular
Meas. of Central Tend.
mean, median, mode
Variability
range, SD, Var.
Transformation
mean, SD, standard score
A normal distribution
Component 3, the linkage between components 1 and 2
Random sampling
Simple sampling: with and without replacement
Sampling Distribution
Distribution of a sample mean
Statistical estimation
1. A big sample distrib. to estimate a pop. distribution.
2. A sample mean to estimate a pop. mean

Figure 3.2

The Content Outline Flowchart of the Unit tendency, and measures of variability are included in the unit on the basis that they are fundamental underlying principles in the statistical structure. The concept of score transformation is included on the basis of its importance in statistics; statistical data generally can be transformed into a form which is convenient to handle, for instance, any normal distribution can be standardized. The concept of score transformation also helps to illustrate the properties of some descriptive measures of the data characteristics, such as, a mean and a variance.

The concept of a normal distribution is selected from various types of frequency distributions in order to emphasize its importance in describing the distributions of a population or a sample, and also because of its role in the central limit theorem.

To illustrate the concept of random sampling distribution, the central limit theorem or a distribution of a sample mean is selected to be included in the unit. The central limit theorem is a basic idea in understanding the concept of a distribution of a statistic. Moreover, the concept of the central limit theorem will also help in understanding the concept of statistical estimation, in particular, using a sample mean to estimate a population mean.

In summary, the unit is intentionally planned as a coherent program of instruction with the following content outline:

1. Population and samples
2. Simple random sampling with and without replacement
3. Frequency distribution which includes:
    - Frequency distribution table

- A histogram
- A frequency polygon
- Some shapes of the frequency distributions
- Using a sample distribution to estimate a population distribution.

4. Measures of central tendency which include:
   - mean
   - median
   - mode
5. Measures of variability which include:
   - range
   - variance and standard deviation
6. Score transformation
   - the nature of a mean and a standard deviation in transformed scores
   - the standard scores
7. A normal distribution
8. Central limit theorem
   - A distribution of the sample means taken from a normal population
9. Point estimate
   - A sample mean as the estimate of a population mean.

Prerequisites

The unit in basic inferential statistics employed in this study, like most other mathematics courses, depends on some previous mathematical knowledge for the learning of the various concepts included in the unit. As a justification for the units, and also a guide for the teacher, the students' prerequisite mathematical knowledge or skill is analyzed.

By investigating the content outline for the unit in the previous section, and considering how deep and wide a scope of topics it is intended grade nine students should cover, the prerequisites can be determined. This unit is intentionally developed in such a way that it contains as few prerequisites as possible, each of which needs no substantive background, so that the teacher can take a short time and insert a required concept during the course of study. The following are considered as the prerequisites:

1. Skill in numerical manipulation such as adding, subtracting, multiplying, dividing, and finding a square root. This prerequisite is fundamental to most math courses. Although the mathematics course outline for Alberta junior high schools has been investigated and it has been judged that grade nine students should have sufficient experience and skill in the use of mathematical fundamental process, this prerequisite certainly needs to be assessed by the teacher before using the unit.

2. Graphing skills: Statistics by nature uses a lot of graphs to illustrate concepts. Grade nine students need both familiarity with the concept and skill in reading, interpreting, and constructing various graphs.

3. Intuitive concepts of probability: This unit does not require a formal knowledge of probability as a major prerequisite, as previously explained in the assumption. However, there may be

some topics or concepts related to probability ideas that the teacher can explain to the students in an intuitive way. For example, in sampling, a score with the most frequency in a given set of scores is more likely to be included in a random sample than one with lesser frequency.

Specific Objectives of the Unit

In relation to recommended objectives of introductory statistics courses, the Committee on the Undergraduate Program in Mathematics (1972) states that:

> When one considers the variety and extent of the demands for an "ideal" first course in statistics, one recognizes the impossibility of having any one course come even close to the ideal . . . (1972, p. 3).

However, the Committee does present guidelines for setting objectives for the course. One such guideline is:

> The introductory statistics course must have limited objectives. Otherwise, it is likely that none of the objectives will be met adequately (1972, p. 3).

Similarly, the specific objectives of this unit must be determined in order to have central guidelines for the teaching, learning and evaluating process during the course of study. Following are eight objectives derived from the content outline of the unit.

The students who study the unit should be able to adequately learn the following concepts:

1. A population and a sample
2. Simple random sampling with and without replacement
3. How to organize data, and how to describe and measure some important characteristics of a set of data

4. The nature of transformed scores
5. The basic idea of a normal distribution
6. The basic idea of the central limit theorem
7. The idea of using some characteristics of a random sample to estimate those of a population:
    - a sample distribution to estimate the shape of a population distribution
    - a sample mean as a point estimate of a population mean
8. How to apply the statistical concepts in the unit to some unfamiliar situation.

INSTRUCTIONAL ANALYSIS

Once the content analysis has been made, and a set of statistical topics to form the intended structure has been chosen to be included in the unit, then the problem of how to communicate these concepts to the students must be analyzed. The instructional analysis is subdivided into three main parts. First, the general instructional approach is described, introducing the main theme for the instructional variable in the unit. Secondly, a more specific instructional procedure to be used in the unit is analyzed. And thirdly, the plan and the description of the intended lessons are presented.

General Approach

Practically everybody possesses some degree of mathematical intuition—grade nine students are no exception. What intuition is, and how it relates to learning, were described at length in Chapter 2 and will not be repeated in detail again.

Intuition, according to Bruner (1960), is "the intellectual technique of arriving at plausible but tentative formulations without going through the analytic steps by which such formulations would be found to be valid or invalid conclusions" (p. 13). He suggests that the nature of intuitive thinking should be recognized and made use of in education, in particular to establish an intuitive understanding of materials before the students are exposed to more traditional and formal methods of deduction and proof. He emphasizes that learning and teaching start from the intuitive level for not only a young child but for anybody approaching new knowledge for the first time. In his own words:

> . . . learning and teaching must start from some intuitive level. This may be true not only of young children entering the educational establishment for the first time, but of anybody approaching a new body of knowledge or skill for the first time. Only a romantic pedagogue would say that the main object of schooling is to preserve the child's intuitive gift. Only a foolish one would say that the principle object is to get him beyond all access to intuition, to make a precise analytic machine of him (1973, p. 83).

Since the unit in basic inferential statistics is considered to be a new body of knowledge approached by grade nine students for the first time, the intuitive and informal method was chosen as the general instructional approach to be employed in the unit. The statistical concepts are illustrated by using examples along with graphs, and the students are expected to do some experiments illustrating appropriate concepts. Teachers are to use an informal and inductive procedure to present the intended concept.

Besides being useful as a general instructional approach, the intuitive fashion is justified for use within the unit according to

Bruner's point of view as already described. The unit is intended to emphasize the structure of basic inferential statistics without heavily relying on probability theory, but rather, the intuitive experience of the concept. To this end, the intuitive approach is an appropriate means of communication. Two influential reports, the Committee on the Undergraduate Program in Mathematics (1972) and the National Advisory Committee on Mathematics Education (1975), strongly advocate the idea of not putting too much emphasis on probability, but rather on basic statistical concepts. The first report recommends that the main objective should be to promote an understanding of basic statistical concepts, rather than extensive study of probability theory or the theory of statistics. The second report recommends that statistics has its own mathematical feature to be presented along with mathematical method—it does not have to be just the application part of probability theory as seems to be the belief in many curriculum developments. The recommendations by these two reports imply the use of an intuitive approach to carry through the instructional procedure of the intended statistical concepts.

Instructional Strategy

The instructional variable is very important to the development of an instructional unit of any selected content. The general intuitive instructional approach of this study as described in the previous section, was presented in general terms as the indicated type of instructional variable to be used. For facilitating the development of the instructional materials in the unit and also the guidelines for the teachers in using those materials, specific instructional methodology or instructional strategies which are along the lines of the selected general instructional approach were designed. The attempt is made to describe the instructional strategies employed in this study. However, one has to recognize that it is almost impossible to single out every instructional strategy employed in developing the instructional materials of the whole unit, or every instructional strategy used by the teachers throughout the long session of teaching the unit. Therefore, only the main instructional strategy employed in this study will be described.

The rationale for attempting to describe the main instructional strategy employed in this study can be viewed as an experimental control on the instructional variable. The instructional variable is an important input to the study and essentially determines the outcome of the study. Therefore, the main instructional strategy employed should be made as clear as possible in order that tight replication of the study be possible in the future.

The designated instructional strategy employed in this study is described by using the ideas and preliminary taxonomy of Merrill and Wood (1974). However, the ideas and technical terms proposed by the two writers need clarification in various stages before the main instructional strategy can be fully defined. The ideas and technical terms will be developed and discussed first by relating them to the context of the instructional unit employed in this study because then one can understand the actual meanings of those ideas and technical terms particularly as they are used in this study.

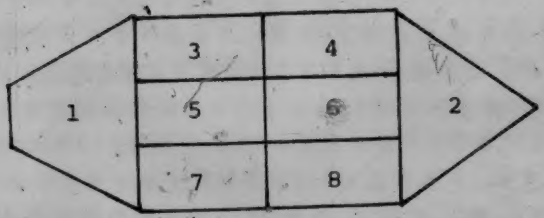
An instructional strategy, according to Merrill and Wood

(1974), can be characterized as consisting of two or more instructional displays in a specified sequence. The definition of an instructional strategy suggests two major classes of variables: (1) display characteristics which concern the characteristics of a given instructional display, and (2) interdisplay relationships which concern the relationships between one display and another display that form a sequence. Display characteristics will be illustrated first, followed by interdisplay relationships.

Display Characteristics

Display characteristics are defined in terms of variables and their associated parameters. Each individual display consists of eight display variables, each of which contains various parameters. Figure 3.3 illustrates the names and the assigned locations of those variables and their parameters in the model of a display.

From this model of display characteristics, one can identify an instructional display as the presentation of those eight variables with appropriate parameters. The eight display variables can be divided into two categories: the content oriented variables and the learner oriented variables, or the input variables and the output variables. They are in pairs as (1,2); (3,4); (5,6); (7,8) as shown in the model. The eight variables in the form of designated pairs, along with some of the parameters employed in the unit, will be illustrated by using examples related to statistics in order to show their defined meanings as they apply to this study.



1. Content Type [Identity (ID); Concept (C); Rule (R)]

2. Response Conditions [Discriminated Recall (DR); Classification (CL); Rule Using (RU); Rule Finding (RF)]

3. Content Mode [Generality (G); Instance (eg or $\overline{\text{eg}}$)]

4. Response Mode [Expository (E); Inquisitory (I)]

5. Content Representation [Enactive (O); Iconic (P); Symbolic (S)]

6. Respond Representation [Enactive (O); Iconic (P); Symbolic (S)]

7. Mathemagenic Prompting [None (no); Mnemonic (mn); Attribute Isolation (ai); Algorithm (al); Heuristic (he)]

8. Mathemagenic Feedback [None (no); Right/Wrong Knowledge of Results (r/n); Correct Answer Knowledge of Results (ca); Mnemonic (mn); Attribute Isolation (ai); Algorithm (al); Heuristic (he)]

"Display Characteristics"
(Merrill and Wood, 1974, p. 15)

Figure 3.3

Instructional Model

1. Content Type: In this study only the word "concept" is used. Actually all three parameters of the content type, ID, C and R, are also included in the content of the unit as illustrated:

1.1 Identity: $\overline{x}$ is considered as Identity because it associates on a one to one basis with the word "mean."

1.2 Concept: "Sample" is a concept because it refers to a collection of any non-empty subsets of a population.

1.3 Rule: "A rule is an ordered relation consisting of a set of domain concepts, an operation, and a set of range concepts" (p. 22). For example: "mean is equal to the total sum of a set of data divided by the total number of data" is a rule because the total sum of data (Σx) and the total number of data (N) belong to a set of domain concepts; $\frac{\Sigma x}{N}$ is a procedure for describing one concept by means of others—therefore is an operation; and "mean" is the element of a set of range concepts.

2. Response Conditions: This variable refers to the logical justification of the learner's response based on the conditions surrounding the request to respond.

2.1 Discriminated Recall (DR): To identify the name for each given identity as in, "Write the name for each of these statistical symbols: $\overline{x}$, Md., s."

2.2 Classification (CL): To identify the class membership as in, "Which one of the following: mean, variance, range measures the central tendency?"

2.3 Rule Using (RU): Referring to the definition of a rule, RU is to find the range concept when a domain concept and an operation are given, as in the example: "Find the mean of a set of data (1, 2, 3, 4) where the mean is the same as the average."

2.4 Rule Finding (RF): Referring again to the definition of a rule, RF is to find an operation when domain and range concepts are given or known, as in the example: "Given a set of data, 1, 2, 3, 10 and the median is 2.5, show how to get that median."

3. Content Mode: "All organized subject matter consists of referents which have been grouped into classes such that these classes are or can be related by propositional statements" (p. 35) is the definition of the content mode. Referents are objects, events or symbols and propositional statements are sentences that relate concept labels by means of relational concept labels. Content mode is recorded either as a propositional statement called a generality (G), or as a presentation of the referent, or some high fidelity representation of the referent, called an instance (eg).

3.1 Generality (G): is illustrated by the following examples:

a. For a concept: "A sample is any nonempty subset of a population."

b. For a rule: "To find the standard deviation of a set of data, work through the following steps: . . . ." (with each step stated in general form).

3.2 Instance (eg) is easy to illustrate by showing an example for either a concept or a rule.

4. Response Mode: Generalities or instances can be represented in either an expository (to tell) or an inquisitory (to ask)

mode. To present a generality or an instance to the student without soliciting an overt response is an expository presentation; on the other hand, if the display is to solicit an overt response from the student then it is an inquisitory presentation.

5. Content Representation: According to Bruner (1966), learners can internalize the representation of the world in three distinct ways: enactive, iconic, and symbolic.

In this unit, doing experiments, participating in completing graphs or examples, along with answering questions to illustrate the intended concepts, are the enactive representations (O). The idea of using graphical figures to illustrate the concept is the iconic representation (P). The use of language and symbols are the symbolic representations.

6. Response Representation: A certain response representation is required on the part of the student and, similar to the content representation, the learner uses enactive (O), iconic (P), or symbolic (S) representations in the response.

In this unit, students are asked to do experiments, as in the case of Lesson 15, and they respond by performing an experiment as an enactive representation. In the same way, during the course of study the students often respond by using iconic and symbolic representation.

7. Mathemagenic Prompting: It is some kind of information added to the content to help the learner.

> Mathemagenic prompting . . . consists of information which is added to the content information for the express purpose of directly or indirectly facilitating a learner's processing of the content information (p. 54).

Mathemagenic Prompting consists of five parameters, each of which can be found during the study of the unit. None (no) indicates no information added to help the student; this sometimes happens in the course of teaching. Mnemonic (mn) is some form of memory aid thought for later recall, such as, the teacher may say, "to memorize the measures of central tendence think of: mean—average; median—middle; mode—peak." Attribute Isolation (ai) is the use of attention focusing devices emphasizing the attributes of a concept. For example, the teacher uses fulcrums to mark the locations of mean, median and mode on the same histogram to isolate the sensitive properties of the three measures. Algorithm (al) is a step-by-step analysis of the problem solving process. For example, the teacher suggests the procedure for solving the problem: "To describe the differences between two sets of data you should look at (1) the distribution, (2) mean, and (3) standard deviation." Heuristic (he) is an incomplete algorithm. As in the example of an algorithm, the teacher may say, "To describe the differences between two sets of data, would it be enough to only look at the means?"

8. Mathemagenic Feedback: The idea is to provide the information after the learner has responded, while "mathemagenic prompting" is to provide information prior to the learner's response. Therefore, those parameters which "prompting" and "feedback" have in common will not be illustrated again; they contain similar ideas for providing information. The only two parameters left are Right/Wrong Knowledge of Results (r/w) and Correct Answer Knowledge of Results (ca). The first parameter is to provide the message which indicates whether the

learner is right or wrong after his response without any additional information, while the second parameter (ca) is to provide a correct answer to the learner after a wrong response or no response. Both parameters are used in this unit and quite often are used together.

Interdisplay Relationships

The other major class of variables in the definition of an instructional strategy is the interdisplay relationships. The interdisplay relationship is categorized into three major variables: sequence, quantity, and qualitative relationship.

Sequence: This variable includes two types of sequences when one thinks of the development of the instructional unit. The first one is the sequence of selected concepts to be presented. It is the nature of mathematics that some concepts need prerequisites, so they have to be presented in the appropriate order. This type of sequence is more content oriented and partly illustrated in the description of the intended lessons. The second type of sequence is a sequence of instructional displays to be presented which moves toward the intended concepts. This type of sequence is directly involved in an instructional strategy. In this unit, the intended sequence of displays to be used as the main instructional strategy is as follows:

1. Display examples or experiments of a concept or a rule. The teacher may start with an introduction of a concept, then illustrate it with some examples or experiments along with the explanation, have students perform some activities, ask leading questions, and so on toward clarifying the attributes of the intended concept.

2. Display a concept or a rule in a general form. This stage

is considered to be the conclusion of what the students have learned from the previous stage. The concept is illustrated verbally or symbolically in a general form by the students with the help of the teacher and the previous examples of the concept.

3. Display exercises or activities for a concept or a rule. After the students presumably have already attained understanding of a concept or a rule in a general or abstract form, this stage will emphasize or evaluate the student's understanding of the concept.

From the main instructional strategy as described above, one can take the view that the first stage is the presentation of a subset of a concept or a rule which is equivalent to instances of a concept or a rule. The second stage is the presentation of the generality of a concept or a rule, and the third stage is the presentation of some instances of a concept or a rule again but with a different purpose than the first stage.

Quantity: This variable indicates the number of displays selected in a particular stage or a whole sequence of an instructional strategy, such as the number of the examples to be presented. The number of instructional displays, generally, can not be appropriately predetermined such that it adequately applies to the instruction of every concept or rule. However, the main instructional strategy in this unit with the approximated value selected for the quantity variable was as follows:

1. To display about 1-4 examples or experiments of a concept or a rule.

2. To display about one general form of a concept or a rule.

3. To display about 1-5 exercises or activities for a concept or a rule.

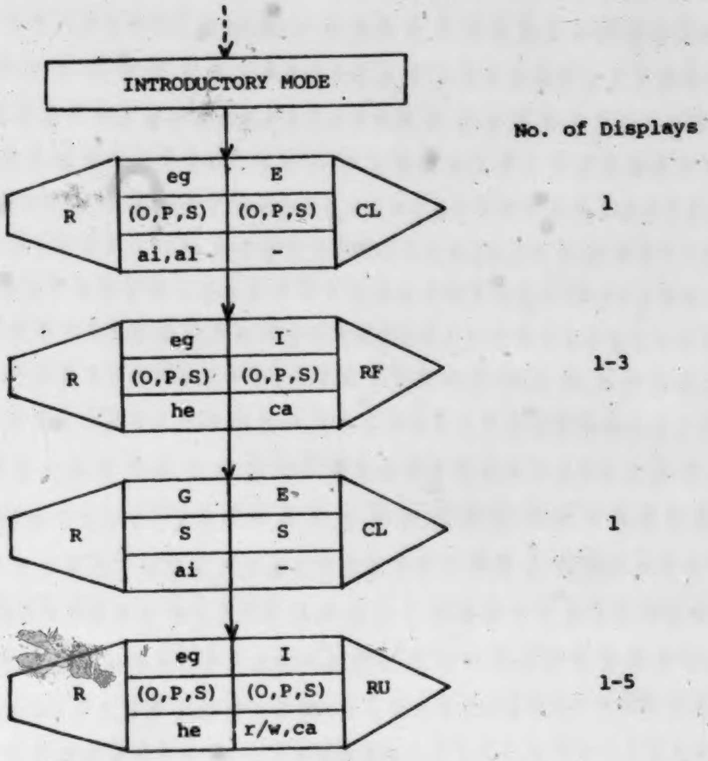
Qualitative Relationship: This variable refers to the quality of an instructional strategy which is composed of any possible various displays incorporated with any possible combination of their parameters. Several research studies have been done on strategies in teaching concepts, such as those by Ginther (1966), Rollins (1966), and Rector (1968), but there was no common conclusive result that one particular strategy is more effective than the others. Henderson (1970) sums it up:

> One can conclude from these findings that a teacher is largely free to choose among the various possible strategies in teaching a concept in mathematics (p. 195).

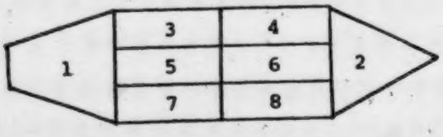
Evidently, the qualitative relationship of the main instructional strategy employed in this unit does not seem to be less effective than others. In addition, the selected instructional strategy gets along well with the general intuitive approach of the unit, and this strategy is also well known among authors, for instance the authors of the series Statistics by Example put out by The National Council of Teachers of Mathematics.

Main Instructional Strategy of the Unit

The description of the two major classes of variables, display characteristics and interdisplay relationships, for constructing the main instructional strategy employed in this study has been completed. In conclusion, the main instructional strategy selected is illustrated in Figure 3.5, using the taxonomy of Merrill and Wood.


INTRODUCTORY MODE
No. of Displays
R
eg
E
(O,P,S)
(O,P,S)
ai,al
-
CL
1
R
eg
I
(O,P,S)
(O,P,S)
he
ca
RF
1-3
R
G
E
S
S
ai
-
CL
1
R
eg
I
(O,P,S)
(O,P,S)
he
r/w,ca
RU
1-5

The Legend of the Symbols
Identified by the following specified
Locations and Descriptions



1. Content Type
   R: Rule
2. Response Conditions
   CL: Classification
   RF: Rule finding
   RU: Rule using
3. Content Mode
   eg: Instance
   G: Generality
4. Response Mode
   E: Expository
   I: Inquisitory
5. Content Representation
   O: Enactive
   P: Iconic
   S: Symbolic
6. Response Representation
   The same as "5"
7. Mathemagenic Prompting
   ai: Attribute isolation
   al: Algorithm
   he: Heuristic
8. Mathemagenic Feedback
   ca: Correct answer knowledge of results
   r/w: Right/wrong knowledge of results

Figure 3.4

The Main Instructional Strategy of the Unit

Plan and Description of Intended Lessons

The instructional plan for the selected topics includes fifteen lessons, each of which contains the following outline: (1) Purpose of the lesson, which emphasizes the objectives the students are intended to achieve; (2) Definition of the experiment, which specifies the general idea of its nature; (3) Specification of materials used in the lesson; (4) Procedure, which contains the instructions for the whole lesson, the directions for doing experiments (if any), and leading questions or remarks to help the students grasp the intended concepts.

The fifteen lessons are also categorized into the two processes of mental activity, assimilation and accommodation. According to Piaget, the process whereby the learner fits every new experience into his pre-existing mental structure is assimilation, and the process of perpetual modification of mental structure to meet the requirement of each particular experience is accommodation. The lessons in which the students are intended to learn new concepts belong to the category of assimilation, while the lessons in which the students are intended to use or practice the learned concepts belong to the category of accommodation. Figure 3.5 illustrates how the fifteen lessons fit into these two categories.

The arrow indicates the direct path from particular assimilaton lesson(s) to the equivalent accommodation lesson(s). Lesson 4 belongs to both categories since its purpose is to introduce new concepts and it also serves as the accommodation lesson for Lessons 1, 2, and 3.

Grade nine students are still considered to be young students

Assimilation	→	Accommodation
Lesson 1, 2, 3, 4	→	Lesson 4, 5
Lesson 6	→	Lesson 7
Lesson 8	→	Lesson 9
Lesson 10	→	Lesson 11
Lesson 12, 13, 14	→	Lesson 15

Figure 3.5

Lessons Categorized as Assimilation or Accommodation who need real involvement and motivation during the process of studying and learning various concepts. The plan is made to have them do experiments in some assimilation lessons and accommodation lessons, and also to have them produce the data by themselves to work with, as in the case of Lesson 4 (drawing samples from the given boxes) and Lesson 5 (collecting their own personal data from the whole class). For the purpose of time saving in producing new data, those data collected in Lessons 4 and 5 are also used in later accommodation lessons, as those of 7, 9, 11, and 15.

The following is a brief description of the intended lessons. The complete set of lessons is found in Appendix A.

Lesson 1 aims at having the students make sense out of a set of data by constructing a table of a frequency distribution. Lesson 2 is continued from Lesson 1 to illustrate the concept of a frequency distribution by constructing a histogram and a frequency polygon. As a result of the activities of Lessons 1 and 2, a variety of distribution shapes will occur, some of which are illustrated for the students in Lesson 3. Lesson 4 aims at the idea of drawing a random sample from a certain distribution of a population; then by investigating a random sample distribution the students can make an inference to a population distribution. The concepts of a population, a sample, and a random sampling without replacement are introduced in this lesson. The students are assigned experiments to do to illustrate the situation in order to clarify the intended concepts. For Lesson 5 it is intended that the students collect real data from their own class members and use that data to work with in the exercises.

After a frequency distribution, a point description of a set of data is introduced. Measures of central tendency, which include mean, median, and mode, are illustrated in Lesson 6, then are followed by activity or exercises for those concepts in Lesson 7. Lesson 8 deals with the concepts of measures of the variability of a set of data, which include range and standard deviation, followed by activity or exercises in Lesson 9. A set of data sometimes needs to be transformed to a certain convenient form to work with, as in the case of z-scores for a normal distribution. Lesson 10 introduces the concept of score transformation and illustrates how a mean and a standard deviation change. Lesson 11 deals with activity or exercises using the concepts in Lesson 10.

So far, the students should have sufficient prerequisite concepts to study a normal distribution; Lesson 12 illustrates the basic concept of a normal distribution by means of a created example. Lesson 13 introduces the basic concepts of the central limit theorem by using the experimenting procedure to convince the students of the concept. Incorporated with Lesson 13, the concept of using a random sample mean as the point estimate of a population mean is introduced in Lesson 14. And as the conclusion of the inferential statistics idea in the unit, Lesson 15 aims at the students applying various learned statistical concepts to both a created experimenting situation where the students can check their answers, and somewhat real situations (e.g. their own personal data) where they have to define a sample and a population and the real answers are naturally unknown.

PILOT STUDY

Purpose

After the content and instruction were analyzed and the first development of the instructional unit was made by the author during September to December 1975, the pilot study was conducted in January 1976. The general purposes of the pilot study were: to observe the initial reactions of grade nine students to the unit, to judge the appropriateness of the instructional materials in order to make improvements in the final development of the unit and to prepare for the actual instructional operation of assessing the feasibility of using the unit.

Specifically, the pilot study was to judge or assess the following:

1. How appropriate is the general procedure of instruction as designed to be used in the pilot study?

2. How well are the instructional materials organized? And is the supply of instructional materials sufficient in both quality and quantity for the students and the teachers?

3. Approximately how much instructional time does a lesson take?

4. Did the students who participated in the pilot study seem to have the prerequisites?

5. What were the students' reactions to the unit of instruction in the pilot study?

6. How is it best to organize a class and form groups of students for performing experiments in the lessons?

Procedure

Eleven grade nine students, five boys and six girls, from a junior high school in Edmonton participated in the pilot study. The study consisted of four forty-minute periods with two consecutive periods per week. A subset of the first development of the unit, Lessons 1, 2, 4, 6 and 13, was used. These lessons were selected for the pilot study on the basis of: (1) the availability of teaching time—four forty-minute periods—in the school, (2) a minimum of prerequisites for each lesson and (3) the lessons' representativeness in relation to the unit. Lessons 1 and 2 contain the necessary concepts for going on to other lessons and they also represent the first five lessons, except Lesson 4, in which students have to do experiments. Lesson 6 is a typical lesson for those from 6 to 12 which makes use of examples and graphs to illustrate the intended concepts. Lessons 13 and 4 represent the lessons employing experimenting procedures. Lessons 1, 2 and 4 were taught by a professor; Lessons 6 and 13 were taught by a regular school math teacher. The author acted as a teaching aid and an observer. There was no test at the end of the pilot study.

The general procedure of instruction employed in the pilot study was as follows. Every student had a set of materials for each lesson which included the sequences of figures and examples duplicated for that lesson so that the teacher could refer to and have individual students participate in drawing graphs or doing examples for illustrating the intended concepts. For some lessons that are long, the students were exposed to the complete set of examples or graphical figures with the help of a teacher to illustrate the intended concepts.

The teacher used the developed lessons (Appendix A) as a guideline in presenting the intended concepts to the students. Since this unit is not an individualized instructional package, the teacher was expected to use some questions equivalent to those in the developed lessons along with his own explanation to illustrate the ideas.

Results and Modifications

Following are the important observations made during the pilot study with respect to each of its six stated purposes. As a result of these observations, some modifications were made in the final development of the unit and instructional plans incorporated in the feasibility study.

1. The general procedure of instruction employed in the pilot study was appropriate. The teacher who participated in the pilot study liked the idea and the students reacted favorably to the immediate involvement with a lesson hand out. The instructional time was considerably reduced by cutting off some work on the board for the teachers.

Consequently, it was decided for the final development of the unit to include two sets of instructional materials. The first set of instructional materials was referred to as "a teacher copy," which consisted of a set of original lessons intended for the unit development. The second set of instructional materials was referred to as "a student copy," which consisted of a set of lessons roughly equivalent to the teacher copy, section by section, but including also graphs and examples which the students are required to draw or complete themselves in order to get involved in learning various concepts and principles.

To proceed with the above idea, the set of lessons in the first development of the unit was slightly modified, such as adding some more activities in Lesson 2 and some instructions in Lesson 14. (A complete set of a teacher copy of lessons can be found in Appendix A.) Then a student copy (the exemplars of the student set of lessons appear in Appendix B) was developed along the same lines. Special attention in developing a student copy was focused on the clear and appropriate duplication of graphical figures, tables, examples and some necessary information to help lead the students toward understanding the intended concepts.

2. When doing experiments requiring the drawing of data from a box, approximately 4 students or two groups of two for a box was sufficient. The pilot study showed that it did not take much time, as in the case of drawing data in Lessons 4 and 13. Moreover, a group of two is convenient to form, generally based on the criterion of friendship, and both students in a group have real involvement in the experiments—one shakes the box and draws data while the other records the results. Consequently, for the final development of the unit and the incorporated plan for the conduct of the unit feasibility assessment, the materials for use in doing experiments, as in Lessons 4, 13, 14, and 15, were developed in the ratio of a box per four students or two groups of two.

3. From observations made during the pilot study, each lesson should be finished within the instructional time of a forty-minute period. Lesson 1, part of Lesson 2, and Lesson 4 were finished within

two consecutive forty-minute periods. Lesson 6 took a little more than forty minutes. For Lesson 13, the students completed the experimenting part of the lesson in 20 minutes. The rest of the time in the period should be enough to construct graphs and discuss the concepts.

Although the instructional time for each lesson could be one forty-minute period, the actual teaching situation in the conduct of the study for the unit feasibility assessment was not restricted to such a period of time. It was left to the teacher's discretion to go faster or slower according to the circumstances.

4. As observed from the pilot study, the students seemed to have sufficient prerequisites. They seemed to handle the questions concerning probability, as in the case of Lesson 2, quite well and seemed to have sufficient knowledge of graph reading and constructing. However, the students' skill in the use of the fundamental arithmetic operations could not be judged during the pilot study because the lessons employed did not involve much computation.

The grade nine students who participated in the pilot study, according to the teacher, already had some experience in probability and graphs. This brings about the idea that the students' prerequisites could be assessed by the teacher. Consequently, regarding the assessment of the students' prerequisites for studying the final unit, it was decided that the prerequisites in probability and graphs would be judged by the teacher for his or her own group of students. However, a standardized test would be used to assess the students' numerical skills.

5. It was noted that generally the students reacted

enthusiastically to the immediate involvement with the new instructional handouts. However, two consecutive periods of instruction bored some students, in particular, during the second period. Besides the length of instructional time, the lack of continuity between the lessons, such as between Lesson 6 and Lesson 13, might have caused a problem.

For the conduct of the unit feasibility assessment, a plan to motivate the students in learning statistical concepts in the unit was made. The students were informed that there would be some prizes for the students who had the best estimates, such as in the major project of Lesson 15. In order to understand the concepts in Lesson 15, however, they had to actively learn the previous lessons.

6. The teacher seemed to have some difficulty in identifying the equivalent sections of the teacher copy and the student copy because the materials were not available to him beforehand. The teacher also needed to do some specific and advanced planning for distributing the instructional handouts to the students and organizing the students to do experiments in some lessons.

Regarding the above concerns, the following plans were made:

The teachers would be given two sets of materials, both a teacher and a student copy, to enable him to use the materials for instructional preparation, such as marking or making notes identifying the corresponding sections between the two copies of materials.

Each student would have a student copy, but not the complete copy at the beginning of the class because it is not an individualized instructional package which the student can study by himself, and because he may lose some part or the whole copy before the end of the study. The students would receive the handouts approximately every time they studied a new concept appearing in that paper.

How to organize in forming a group to do experiments and exchange data among various groups, as in the case of Lessons 4, 13, 14 and 15, would be left to the teacher's own discretion. However, discussions would be held during the teachers' initial in-service training period.

SUMMARY

At the outset of the construction of a unit in inferential statistics for grade nine students, assumptions, or philosophical background, caused the author to believe that such a unit is constructible. The general plan for the unit's construction has been made in the form of major guidelines to follow stage by stage until completion. The first two stages involve the analysis of content and the analysis of how to communicate the content to the students—instruction. The first draft of a set of intended lessons was developed following that analysis. As a formative evaluation for the process of constructing the unit, a pilot study using a subset of the initial lessons was conducted. The results of the pilot study have brought about the final development of the unit, which includes two sets of equivalent lessons, one for a teacher and the other for students, and also incorporates plans for the feasibility study of the unit. The feasibility study of the unit can be viewed as the summative evaluation of the unit. In the next chapter, test instruments employed in this study will be described.

CHAPTER 4

TEST INSTRUMENTS

The threefold purpose of this study has been stated as follows: (1) to construct an instructional unit in inferential statistics, (2) to determine the feasibility of teaching the unit to grade nine students, and (3) to investigate and compare the students' anchoring ideas about some selected statistical concepts before and after the instruction of the unit. The construction of the instructional unit has been described in Chapter 3. In the present chapter, the test instruments employed in both investigating the students' anchoring ideas, and assessing the feasibility of teaching the unit, are described. The first section of the chapter deals with the test instrument called the "Conception Test"—a test developed to be used as both a pre- and a post-test in investigating the students' anchoring ideas about some selected statistical concepts. The second section deals with the Achievement Test, the Skill Test, the Student Reaction Test, and the Teacher Reaction Test—instruments used in assessing the feasibility of teaching the unit to grade nine students.

CONCEPTION TEST

The Conception Test is described under the following headings: the rationale for the test, the test construction, the procedures used in scoring, the reliability and the validity of the test.

Rationale

Ausubel (1968) makes a statement similar to the one that has already been mentioned in relation to the theoretical background discussed in Chapter 1:

> . . . One obviously important variable affecting the learning and retention of new, logical meaningful material is the availability in cognitive structure of specifically relevant anchoring ideas at a level of inclusiveness appropriate to provide optimal relatability and anchorage . . . (p. 131)

It is reasonable and logical to assume that the students, before studying the unit in inferential statistics possess some type of anchoring ideas about statistics in cognitive structure. These anchoring ideas may be right or wrong or incomplete with respect to the nature of the concepts selected for this particular unit of instruction. However, if the lessons on inferential statistics in the unit are prepared in such a way that they are logical meaningful materials that appropriately relate to those prior anchoring ideas, then the instruction of those lessons in the unit should result in the students having different anchoring ideas from the previous ones about the same set of statistical concepts. During the instruction of the unit, the students' anchoring ideas are expected to improve in two directions—quantitatively and qualitatively.

The idea of investigating anchoring ideas before and after the instruction of the unit originated with the consideration of Ausubel's theory. It should be noted that anchoring ideas are more inclusive and general than prerequisite skills and knowledge.

An anchoring idea, according to Ausubel, is an idea related to some learning task that is at a higher level of abstraction, generality, and inclusiveness than the learning task itself. In relation to the learning tasks involved in basic inferential statistics, an anchoring idea is the student's idea or knowledge, at whatever level or in whatever form, about various statistical concepts. It is assumed that at any stage of time the student would exhibit some observable behavior reflecting his anchoring ideas in a testing situation created to assess his knowledge about selected statistical concepts. The expected observable behavior would be in the form of either oral or written verbal expressions.

Test Construction

Now that the rationale has been described and presumably accepted, that is, that the students do have prior anchoring ideas about statistical concepts and that those anchoring ideas can be investigated by using some appropriate type of test, the design and construction of such a test are considered as follows:

The Concept Selection. The first stage of construction was to investigate statistical concepts to be covered in the test. Two main criteria were set in selecting the concepts to be included. First, the selected concepts must relate to, or come out from, the content of the instructional unit, because the main theme of this study concentrates on the effect of the instruction of such a unit on the students' anchoring ideas. Second, the selected concepts must be readily communicable to the students, especially in the pretest. Special attention was given to the second criterion in developing the test, since it is more difficult to meet than the first.

Objectives of the Test. Within the guidelines of the above two criteria, certain statistical concepts were chosen to be included in the test. These selected concepts are presented in the form of objectives. These objectives will be useful in designing the questions in the test and also in marking the results of the test. The objectives of the Conception Test are as follows:

To discover the students' anchoring ideas about the following statistical concepts:

1. Measures of central tendency
2. Variability of scores
3. Transformed scores
    - 3.1 the effect on "mean" when:
        - a. each of the scores is added to a constant
        - b. each of the scores is multiplied by a constant
    - 3.2 the effect on "variance" when each of the scores is added to a constant
4. A normal distribution
5. Estimation
    - 5.1 the use of a sample mean to estimate a population mean
    - 5.2 the effect of using a mean from a different size sample to estimate a population mean.

Test Design. On the basis of these five objectives, an eight question Conception Test was designed. The eight questions were arranged in appropriate order since some questions require information from previous questions in order to be understandable to the students. The complete Conception Test is presented in Appendix D.

The communicability of the Conception Test to the students is another aspect of concern in the test design. Three major areas were considered during the planning of the test: how to establish questions in the test, the use of appropriate terms or language, and the procedure for presenting the test to the students.

In each of the eight questions, examples are used to clarify the situations and definitions so that the students can easily understand what is being asked. The illustrations are followed by specific questions on the designated concept.

Language or terms used in the test could block communication to the students. Statistical terms or language are avoided if possible and common terms or language are used in their place. For example, a "middle" score is used to represent any measure of central tendency for a set of scores.

The partial interview procedure was considered to be the procedure for presenting the test to the students. This procedure was proposed in order to make sure that the students would understand the instructions and the questions, especially in the pretest. Moreover, the interviewer would have the opportunity to ask for more information from the students if their answers were incomplete.

Test Administration Plan. In preparation for the administration of the eight question Conception Test, each question was typed on a separate piece of five by eight inch paper. The actual administration of the test proceeded as follows:

1. The interviewer began by trying to ease the student's tension and explaining the nature of the test by using such comments as "This test is not concerned particularly with right or wrong answers. I am interested in what you think, or your ideas about the questions. So feel free to express your own ideas." This introductory mode is important, especially for the pretest.

2. Each question was handed to the student to read, then he or she answered orally. There was no time limit.

3. The student's answer was recorded on tape, or on a piece of paper prepared by the interviewer.

4. The interviewer sometimes had to make suggestions or answer the student's questions in order to help him or her understand the instructions. However, certain key words which are a part of the concept being tested were not explained, for instance, the "middle" score used as a measure of central tendency. In such instances the interviewer's answer should be something like, "Whatever you think it is" or "That's what I am interested to know."

5. The interviewer sometimes asked the student to clarify his answer.

Administration of Pilot Test. In order to judge the appropriateness of the communicating procedure, the amount of testing time, and to a certain degree determine the test validity and reliability, a pilot Conception Test was administered in December 1975. The participating students were interviewed by a professor, and the author acted as the observer.

The amount of testing time for a student was approximately twenty minutes. The general communicating procedure was relatively appropriate. Only a few words had to be rewritten, and some specific questions were added to various test question items, such as, "Why or why not?" or "How do you get the answer?" in order to help standardize the test instructions.

Test item 6, which involved the illustration of computing a special number (a variance) and then having the student compute the other one by himself before answering the intended question, had to be carefully instructed. Besides this item being time consuming, the student tended to make a computational error which the interviewer had to help correct and the student seemed to think that the computational result was the end of this test item. The interviewer, therefore, had to point out the question intended to be answered by the student.

Scoring Procedure

The purpose of the Conception Test is to investigate the availability of the students' anchoring ideas about certain statistical concepts at two different stages of time—before and after the instructional unit. Generally, the analysis of the students' answers is not to score but to summarize what those ideas are. Both pretest and post-test employ the same procedure:

1. Quantifying the answers: To quantify the answers is to list all different answers for each question item from all students who take the test, and to also record the number of students answering each of those different answers. Specifically, it is to fill out a table with two headings: students' answers, and the frequency or the number of repeated answers.

2. Ranking the answers: To rank the answers is to arrange the quality of the available answers from the best to the poorest to the extent that they answer the intended question in the test. This stage is a continuation of the first stage, ranking the best available answer as number one, the second best as number two, and so on. The best quality answer is defined with respect to the objective set for the test, and whether the answer is right or wrong according to the possible solution of that test problem. For example, question one in the Conception Test is intended to investigate the ideas about a measure of central tendency; the possible solutions for this item are the ideas of mean, median, and mode. Actually, some answers, as in the above cases of mean, median, and mode, are equally qualified to be the best answer. However, ranking has to be done in order to show the different types of answers. Question five in the Conception Test is intended to investigate the ideas of normal distribution by asking the students to give examples. The possible solutions should be categorized into four areas: (1) a normal distribution occurring in mathematical fact, (2) a normal distribution occurring in natural

phenomena such as height, weight, and I.Q., (3) a normal distribution occurring in specific cases such as family size in a specific area, which is approximately normally distributed, and (4) others that cannot be justified as a normal distribution. Each student was allowed to give about three examples to illustrate the idea. If half or more of those examples belong to the highest category, then the student's answer is considered to be of a higher category.

3. Combining the students' pre- and post-test answers into a common list. For each question item in the conception test, the lists, as described in the second stage, of all answers for both pre- and post-test results, were combined into a common rank according to quality. The quality of the students' answers normally had to do with the ideas expressed; if the students' answers in pre- and post-tests indicated the same ideas, even though using different terms or means of expression, those answers were combined in the same rank. The number of students who expressed each of those answers in the combined list was still reported separately under the headings of pretest and post-test. The combined pre- and post-test answer list was prepared to make a comparison between the number of students giving each type of the different available answers on pre- and post-tests and also for convenience in coding the students' answers.

4. Coding the students' answers. As described in the third stage, all different answers for each question item were labelled with ranking numbers. In this stage, those ranking numbers were used to record the student's answer to each question item in the Conception Test. For example, one student's answer was recorded

as (3,1) for the question item one. This coding format—(3,1)—indicated that that student's idea was the answer ranking number three in the pretest, and then he changed his idea to the answer ranking number one in the post-test. The first element of the ordered-pair coding number indicates the answer to the pretest, while the second element represents the post-test answer. This coding system was prepared to indicate the change in answers or ideas, occurring for individuals or a group of students with respect to each question item, between pre- and post-tests.

Test Reliability

As with any test instrument, it was necessary to be sure that the Conception Test is reasonably reliable. To that end reliability was thought of as the ratio between variance associated with true individual differences and total variance.

For most test instruments this ratio is estimated by using one of the following three reliability procedures: (1) test-retest, (2) parallel-form, or (3) internal consistency. Unfortunately, due to the nature of the Conception Test, none of these procedures was appropriate.

The Conception Test employed the partial interviewing procedure with individuals or a small group, and involved a lot of testing time. Furthermore, the scoring procedure employed in the Conception Test was not to give a numerical score to each answer to the test; instead, the students' expressed ideas were their answers to the test and these ideas were classified and summarized. Since all three reliability procedures mentioned above require the students' scores in a numerical form to compute the reliability coefficient, none of them could be used to assess the reliability of the Conception Test.

Briefly reviewing these procedures, the test-retest estimate of reliability is obtained by administering a test to a group of students, readministering the same test to the same group of students at a later date, and correlating the two sets of scores. The parallel-form estimate of reliability is obtained by giving two equivalent forms of a test to the same group of students on the same day and correlating these results. Internal consistency procedure obtains reliability estimates from one set of data using several methods. For example, Split-Half and Kuder-Richardson Estimates (KR-20, KR-21). Each of these methods employs numerical data to compute the reliability coefficient. The internal consistency procedure generally estimates the degree of homogeneity of the items in the test, or the degree to which the item responses correlate with the total test scores. But the Conception Test is intended to investigate individual selected statistical concepts.

Although the reliability of the Conception Test could not be estimated by any particular computational procedure, one could assume that the reliability of the test would be high if the possibility of error variance, that is, variation in a person's score due to error, could be reduced. Maximal reliability of the test would be obtained if the variances of the true scores and of the observed scores were equal, or the ratio between the two variances was equal to one. Several sources of error could contribute to inaccuracies in the observed scores, and consequently determine the degree of reliability of the test. Regarding the Conception Test, such possible sources of error are: (1) the testing procedure, (2) the subjects, (3) the conditions, and (4) the scoring method. However, an attempt to control these possible sources of error was made in order to improve the test reliability.

Errors of test measurement due to the procedure mainly involve the test construction and the test administration. The wording of the questions and the questioning procedure, along with the administration of the directions for the test, were carefully planned so that the students who took the test would know equally well what idea or ability they were supposed to demonstrate. Moreover, from judging the results of the pilot test, some modifications were made, such as the addition of some specific questions at the end of certain test questions in order to standardize the instructions for the interviewer, and to clarify the ideas for the students.

Error due to the students could be caused by change and development of the various students during the testing time. The Conception Test, which employed a partial interviewing procedure, involved a lengthy testing time to complete the whole group of students. However, this type of error could be allowed as a part of the results of the study because it also involves internal change or development of the individual student up to a certain point of time. This is especially true because the Conception Test items were not of the type usually found in school mathematics tests.

In order to prevent error due to conditions, the students, individually or in a small group, were randomly selected to take the test in a room remote from any annoyance that might come from the regular class. The students were instructed to sit far from each other when testing in a small group so that they did not influence each other's ideas. They were also instructed not to relate information about the test to fellow students who would be taking the test at a later date.

To control error due to marking, a scoring procedure was designed as a guideline for managing the students' answers. Moreover, as has been mentioned, after the administration of the pilot test some more clarified questions were added to several test items to encourage the students to provide more information with their expressed ideas. The added information would be helpful in the interpretation of those ideas in the scoring procedure. Scoring both pre- and post-conceptual tests was done by one person.

Test Validity

In assessing the validity of the Conception Test, two types of validity, content validity and construct validity, were considered. For content validity, there must be congruence between the item as stated in the test and the concept to be learned as set down in the objectives. For construct validity, there must be evidence that the items as stated in the test measure the hypothetical construct called "anchoring ideas."

The Conception Test was developed with close cooperation between one mathematics education professor and the author. Critiques were done and discussions were held at various stages of the test construction in order to improve the test validity, both in content and construct. Particularly, the procedures involving the selection of statistical concepts, the wording and questioning procedure, the administration of the pilot test and test modifications after pilot testing, were carefully designed and improved.

An "anchoring idea," according to Ausubel's definition, is an idea related to some learning task which is at a higher level of abstraction, generality, and inclusiveness than the learning task itself. There is a problem in assessing content validity with respect to these anchoring ideas since content validity, according to the report of Technical Recommendations for Psychological Tests and Diagnostic Techniques (1954), is evaluated by how well the content of the test samples the set of situations about which conclusions are to be drawn, and content validity is examined when concern is about the type of behavior involved in the test performance.

In this study then, content validity is indirectly important because there is no direct correspondence between the Conception Test items and the set objectives of the unit in inferential statistics.

For construct validity, the report on Technical Recommendations for Psychological Tests and Diagnostic Techniques (1954) has distinguished its characteristics and validation procedure as follows:

> Construct validity is evaluated by investigating what psychological qualities a test measures, i.e., by demonstrating that certain explanatory constructs account to some degree for performance on the test. To examine construct validity requires both logical and empirical attack. Essentially, in studies of construct validity we are validating the theory underlying the test. The validation procedure involves two steps. First, the investigator requires: From this theory, what predictions would we make regarding the variations of scores from person to person or occasion to occasion? Second, he gathers data to confirm these predictions . . . (p. 14)

For assessing the construct validity of the Conception Test, the first step involves the anchoring idea construct in reference to Ausubel's theory which states that:

> . . . The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly . . . (1968, p. vi)

In relation to the unit in basic inferential statistics prepared in this study, one can assume two predictions about the students' anchoring ideas:

1. The students should already have some anchoring ideas about statistical concepts before they study the unit in basic inferential statistics. Among those anchoring ideas, there should be some high quality ones with respect to the selected statistical concepts tested.

2. Since anchoring ideas are important to learning, there should exist some relation between the learning outcome and

the quality of anchoring ideas. Specifically, the quality of the students' anchoring ideas should be related to the students' performance on the Achievement Test (the learning outcome).

For the second step of the validation procedure, data were gathered to confirm the two postulated attributes of the anchoring idea construct or the two hypothesized predictions as described in the first step of the validation procedure. If the evidence did confirm the two predictions derived from Ausubel's theory, then the Conception Test can be claimed to have some construct validity. In other words, the Conception Test did measure the variables of the anchoring idea construct involving statistical concepts if the two hypothesized predictions were supported by the data.

For the first hypothesized prediction, the evidence was examined by looking at the pre-testing results of the Conception Test in which the students reflected their anchoring ideas in a variety of test performances. Some of those observable performances correspond to the correct statistical concepts tested, indicating that some students already had high quality anchoring ideas. Table 4.1 confirms the hypothesized prediction that there existed a number of students who already had correct ideas on each of the selected statistical concepts. However, the number of students who had high quality anchoring ideas varies with each question due to the nature or abstraction and difficulty of that particular statistical concept.

For the second hypothesized prediction (that there should be some relationship between the quality of the students' anchoring ideas and the students' performance on the Achievement Test) the

Table 4.1

The Evidence of the Existence of Anchoring Idea Construct

Statistical Concepts Pretested	Number of Students With Correct Ideas From 42 Students
1. Measures of central tendency	28 (66.67%)
2. Variability of scores	2 (4.76%)
3. Transformed scores	
3.1 The effect on a mean when:	
a. each of the scores is added to a constant	10 (23.81%)
b. each of the scores is multiplied by a constant	17 (40.48%)
3.2 The effect on a variance when:	
each of the scores is added to a constant	2 (4.76%)
4. A normal distribution	29 (69.05%)
5. Statistical estimation	
5.1 The use of a sample mean to estimate a population mean	8 (19.05%)
5.2 The effect of using a mean from a different size sample to estimate a population mean	16 (38.10%)

evidence was gathered from the post-testing results of the Conception Test, and of the Achievement Test that measured the same statistical concepts as the Conception Test. The Achievement Test can be viewed as measuring the same variables of anchoring idea construct as the Conception Test because the performance of the students in the Achievement Test reflects the internal anchoring ideas available to the students at that particular time. Therefore one could predict that there would be a high degree of agreement between the results of the two tests.

The degree of agreement between the results of the two tests, with respect to the same statistical concepts, was determined according to three categories:

1. The number of students who expressed satisfactory answers on both tests;
2. The number of students who expressed unsatisfactory answers on both tests;
3. The number of students who expressed satisfactory answers on only one of the two tests.

The first two categories indicated the degree of agreement between the two tests. However, they indicated different effects of the instruction of the unit—the first category indicating a positive effect and the second category indicating a negative one. The third category indicated the degree of disagreement between the two tests. Therefore, if there were more students in the first two categories than in the third category, the second hypothesized prediction was confirmed, or the two testing results did indicate a high degree of agreement.

Table 4.2 indicates the numbers of students in the three categories of relationship between the two testing results on the selected statistical concepts involved. The satisfactory answers from the Conception Test used in the table were defined along the same lines as the items for the analysis of change between the students' pre- and post-test anchoring ideas. The Achievement Test items used as a measurement of comparison along with the satisfactory Conception Test answers were as follows:

With respect to the Achievement Test items illustrated in Appendix C and the statistical concepts post-tested in Table 4.2:

1. At least one correct answer from the Achievement Test items 4, 5, and 6 was used as the satisfactory answer for Concept 1.

2. Only the correct answer from the Achievement Test item 10 was used for Concept 2.

3. Only the correct answer from the Achievement Test item 11.1a was used for Concept 3.1a.

4. Only the correct answer from the Achievement Test item 11.2a was used for Concept 3.1b.

5. Only the correct answer from the Achievement Test item 11.2b was used for Concept 3.2.

6. At least one correct answer from the Achievement Test items 14.1 and 14.2 was used for Concept 4.

7. Only the correct answer from the Achievement Test item 16.4 was used for Concept 5.1.

8. Only the correct answer from the Achievement Test item

Table 4.2

The Evidence of the Existence of a Relationship Between the Students' Anchoring Ideas and Test Performances on the Achievement Test

	Number of Students in		
	Cat. 1	Cat. 2	Cat. 3
1. Measures of central tendency	33 (84.62%)	0 (0.00%)	6 (15.38%)
2. Variability of scores	12 (30.77%)	14 (35.90%)	13 (33.33%)
3. Transformed scores			
3.1 The effect on a mean when:			
a. each of the scores is added to a constant	32 (82.05%)	1 (2.56%)	6 (15.38%)
b. each of the scores is multiplied by a constant	33 (84.62%)	1 (2.56%)	5 (12.82%)
3.2 The effect on a variance when:			
each of the scores is added to a constant	7 (17.95%)	6 (15.38%)	26 (66.67%)
4. A normal distribution	18 (46.15%)	5 (12.82%)	16 (41.03%)
5. Statistical estimation			
5.1 The use of a sample mean to estimate a population	30 (76.92%)	1 (2.56%)	8 (20.51%)
5.2 The effect of using a mean from a different size sample to estimate a population mean	28 (71.79%)	3 (7.69%)	8 (20.51%)

Note:		
	Category 1:	Getting satisfactory answers on both Conception Test and Achievement Test
	Category 2:	Getting unsatisfactory answers on both tests
	Category 3:	Getting satisfactory answers on only one of the two tests

17.3 was used for Concept 5.2.

From Table 4.2 almost every case indicates a high degree of agreement between the two testings. This confirms the second hypothesized prediction.

The confirmation of both hypothesized predictions regarding underlying anchoring ideas indicates the construct validity of the Conception Test. However Concept 3.2 (the effect on a variance when each of the scores is added to a constant) indicated that 66.67% of the students were in the third category—the category of disagreement between the two testing results. In relation to this incidence, Cronbach and Meehl (1955) explain that:

> . . . If two tests are presumed to measure the same construct, a correlation between them is predicted. . . . If the obtained correlation departs from the expectation, however, there is no way to know whether the fault lies in test A, test B, or the formulation of the construct (p. 287).

With respect to Concept 3.2, the disagreement of results may be due to the slightly different statistical concepts measured by the two tests. The Conception Test measures the concept of the transformation of a variance; the Achievement Test item measures the concept of the transformation of a standard deviation. The students did not grasp the relationship between the two concepts tested. More emphasis was put on the concept of a standard deviation than of a variance during the unit instruction. As a result, the students had more correct answers on the concept of the transformation of a standard deviation than of a variance.

TEST INSTRUMENTS FOR FEASIBILITY ASSESSMENT

The assessment of the feasibility of teaching the unit on inferential statistics to grade nine students is determined with respect to three reference domains: (1) the objectives of mathematics for junior high schools in Alberta, (2) the reactions of the students who study the unit, and (3) the reactions of the teachers who teach the unit. Two test instruments—the "Achievement Test" and the standardized "Skill Test"—were used to judge how well the unit on inferential statistics carries out the objectives of mathematics for junior high schools in Alberta. The "Student Reaction Test" and the "Teacher Reaction Test" were developed to test the responses of those two groups. Each test instrument is described in detail below.

Achievement Test

Purpose

The primary purpose of this test is to measure the contributions of the unit of instruction to the objectives of mathematics for junior high schools as listed:

1. To develop an understanding of mathematical concepts and appreciation of mathematical structure.
2. To develop skills in the use of the fundamental process.
3. To develop systematic methods of analyzing problems and of presenting their solution.
4. To develop habits of precise thought and expression.
5. To develop an understanding of the significance and application of mathematics in the modern world.

3

Specifically the instrument called the "Achievement Test" was developed to measure the contribution of the unit of instruction to the first, third and fifth objectives of the above five.

Test Construction

The first task in the construction of the Achievement Test was to describe the levels of mathematical thinking at which the students who took the test were expected to perform. The second task in the construction was to describe the test design in which the levels of mathematical thinking, the three objectives (the first, third and fifth) for junior high school mathematics, and the specific objectives for the unit of instruction were taken into consideration. The third task was to decide on timing and scoring procedures.

Levels in Mathematical Thinking. According to Avital and Shettleworth (1968), there are three levels in mathematical thinking. The first and most basic level is recall or recognition of material in the form in which it was taught. The second and higher level is called algorithmic thinking. This level is the straightforward generalization or transfer from learned material to material that is similar. The third and highest level in mathematical thinking is called open search. Avital and Shettleworth explain that:

> Behavior on this level emerges when the student's repertoire is not confined to operations and the solution of problems for which he has learned a straightforward procedure. He can rearrange or restate the parts of a problem and see among them new relationships which are relevant to the sought-for solution (p. 6).

Avital and Shettleworth emphasize the distinction between the first two levels and the highest level in mathematical thinking; the first two levels are more straightforward procedures, or reproductive thinking, while the open search level is not a straightforward procedure. It is productive thinking.

To compare the above three levels with the categories of Bloom's Taxonomy, recall or the recognition level is equivalent to the level of knowledge; generalization or algorithmic thinking level is equivalent to the level of comprehension and the level of application; and the open search level is equivalent to the levels of analysis and synthesis.

In designing the Achievement Test in this study Bloom's Taxonomy and Avital's thinking were combined and referred to as the following four levels: (1) Knowledge (K); (2) Comprehension (C), (3) Application (A), and (4) Open search (OS).

The levels of Comprehension and Application need further illustration, since both belong to the algorithmic thinking level. The major distinction between these two categories is the amount of novelty in the given task from the learner's point of view. The use of algorithms such as manipulative skills, the straightforward use of formulas to compute the answer, to illustrate given definitions or statements, and to translate from words to mathematical symbols and vice versa, belong to the level of Comprehension. The 'story' problems to which the students have not been exposed at the time they learned the principle are in the category of Application. The general behavior required for the level of Application is the ability to use the given principles to solve problems relating to, for instance, everyday situations or scientific phenomena.

Test Design. The Achievement Test is divided into two parts. Achievement Test Part I is designed to measure the contributions of the unit of instruction to the first and fifth objectives of Alberta junior high school mathematics. Achievement Test Part II is designed to measure the contributions to the third objective. The complete Achievement Test can be found in Appendix C.

For the purpose of specifying the content to be covered in the Achievement Test, the first and fifth objectives of Alberta junior high school mathematics were restated as follows: Obj. 1, To develop an understanding of statistical concepts and structure, and Obj. 5, To develop an understanding of how to apply statistical concepts to unfamiliar situations.

Achievement Test Part I is constructed to consist of 45 test items, and each item is designed as a constructed response-question. The student is expected to produce his or her own answers or solutions, then place them in the provided spaces. Achievement Test Part I is also designed to reflect each of the specific objectives of the unit, which are then categorized under the three levels: Knowledge (K), Comprehension (C), and Application (A). All forty-five test items are used to measure the unit contribution to the first objective of junior high mathematics, but only a subset of eighteen items, categorized under the level of Application, are used to measure the unit contribution to the fifth objective. Table 4.3 shows the test grid design of the Achievement Test Part I, using the following specific objectives of the unit.

Table 4.3

Test Grid of Achievement Test Part I

Spec. Obj.		Test Item Nos. in Specified Categorization			Total
		Knowledge	Comprehension	Application	
1.				13.1,13.2 16.1,16.2	4
2.			12.1,12.2,12.3		3
3.	3.1	1.1,3.1, 3.2,3.3	1.2,1.3,2.1,2.2, 2.3,2.4		10
	3.2	7.1,7.2, 7.3	4,5,6	16.3	7
	3.3	8	10	9	3
4.			11.1a,11.1b, 11.2a,11.2b, 11.3a,11.3b		6
5.				14.1,14.2	2
6.				15.1a,15.1b, 15.2a,15.2b, 15.3	5
7.				15.4,16.4, 17.1,17.2, 17.3	5
Total		8 (17.78%)	19 (42.22%)	18 (40%)	45

The students should be able to learn the following concepts:

Spec. Obj. 1: Population and sample
2: Sampling with and without replacement
3: Descriptive procedure of data
    3.1 Frequency distribution
    3.2 Measures of central tendency
    3.3 Measures of variability
4: Score transformation
5: A normal distribution
6: Central limit theorem
7: Statistical estimation.

In order to construct Achievement Test Part II as an instrument to measure the unit contribution to the third objective of junior high mathematics ("to develop systematic methods of analyzing problems and of presenting their solutions"), this objective was redefined in terms of the unit in inferential statistics as follows: (1) to develop the ability to apply statistical procedure, learned from the unit, to problem solving, and (2) to promote the open-search method of problem solving in light of the study of the unit.

From the above definitions, Achievement Test Part II is designed so that the first two major questions involve the statistical procedures included in the unit content, and are categorized under the level of Application. The last three questions are categorized under the open search level. The following is the outline of Achievement Test Part II. (Complete set in Appendix C.)

Test no. 18 is intended for "Descriptive Procedure"

18.1 is intended for "the use of frequency distribution"

18.2 is intended for "the use of a measure of central tendency"

18.3 is intended for "the use of a measure of variability"

Test no. 19 is intended for "Inferential Procedure"

19.1 is intended for "the use of a sample distribution"

19.2 is intended for "the use of a sample mean"

Test no. 20, 21, 22 are intended for "Open Search Procedure"

Timing and Scoring Procedure. It was originally intended that the Achievement Test Parts I and II would be completed in approximately one hour. However, when the test was used in this study, the students were allowed to take two forty-minute periods, and almost all forty-five students completed the test within fifty minutes to one hour. Only one student finished in one period, and only one stayed until the end of two periods.

In setting the scoring procedure, it was decided that for Achivement Test Part I each of the forty-five test items would be weighted equally, and assigned only one for right and zero for wrong. There is no partial score for any individual test item. The decision to award one or zero has to be made in relation to the degree that the answer approaches the concept tested by that test item.

The first five test items of the Achievement Test Part II employ the same scoring procedure as Part I. Each of the last three open search questions is worth three points. The student's answer to each of the three questions is broken down into three components: (1) defining a sample or samples used in the problem, (2) defining a statistical procedure or method used, and (3) making a conclusion or inference to a population or populations in the problem. Each of the component parts of the student's answer is worth one point. No fractional score is given.

Criterion Referenced Measurement

The Achievement Test in this study is intended for use as a criterion referenced test. The criterion, or a specified level of quality used to judge a student's performance on the test, had to be considered and assigned. The criterion, or the specified level of quality of the student's knowledge about a specific concept in this test, is designated as a certain number of test items with right answers for each of the specific objectives of the unit, and also a certain number for the whole test.

In setting the criterion, or the least number of test items the students are expected to achieve in this test, two factors were considered: the degree of difficulty of the test item, and the degree of importance of the tested concept to the unit. The degree of test difficulty was decided in direct relation to the four levels of mathematical thinking: Knowledge (K), Comprehension (C), Application (A), and Open Search (OS). The higher the level of mathematical thinking required, the fewer right answers to those test items can be expected. Specifically, it was decided to set the criterion with respect to the degree of difficulty in this test as approximately 90-100% of K, 60-66% of C, 50% of A, and 40% of OS.

The degree of the importance of the tested concepts to the unit is considered according to the amount of emphasis the teacher is expected to place on each of the specific objectives of the unit, and the roles those testing concepts contribute to the unit as a whole. Generally, the more important the tested concept is to the unit as a whole, the higher the number of right answers can be expected to those test items. These two factors, difficulty and importance, were considered when assigning the criterion reference for this Achievemet Test with the following results: (for more detail see Table 4.4 and Table 4.5)

1. The criterion reference of the instrument to measure the unit contribution to the first objective of Alberta junior high mathematics is 60%.
2. The criterion reference of the instrument for the fifth objective is 50%.
3. The criterion reference of the instrument for the third objective is also 50%.

Test Validity and Reliability

The Achievement Test was developed by the author, two mathematics education professors had input by criticizing the design of the test grid in relation to its content validity, the representativeness and the reflection of the content in the unit. Wording of the test items and questioning format were also discussed among the

Table 4.4

The Criterion Referenced Measurement for Achievement Test Part I and the Application Part

Spec. Obs.		No. of Test Items K	C	A	Total	Criter. Refer.
1:				4	4	2
2:			3		3	2
3:	3.1	4	6		10	7
	3.2	3	3	1	7	5
	3.3	1	1	1	3	1
4:			6		6	3
5:				2	2	1
6:				5	5	3
7:				5	5	3
Total		8	19	18	45	
Select as Crit.		7 (90%)	11 (60%)	9 (50%)	27 (60%)	27

Table 4.5

The Criterion Referenced Measurement for Achievement Test Part II

No. of Test Items	Category	Total Scoring Weight	Criterion Ref.
5	A	5	3 (60%)
3	OS	9	4 (44%)
Total 8		14	7 (50%)

panel. In addition, both parts of the Achievement Test were validated by correlating the test results from one school in the study to the results of I.Q. tests and the students' previous term math grades. The computed correlation coefficients of the Achievement Test Part I results and the I.Q. scores, and also of the Achievement Test Part I results and the previous math grades from the same group of students were .80 and .90 respectively. The computed correlation coefficients of the Achievement Test Part II results and I.Q. scores, and also the Part II results and previous math grades were .76 and .82 respectively.

All Achievement Test Part I items were scored by using the dichotomous system designated either as a 0 or a 1. The total score for the test items for each student was recorded, as was the total number of students who had a correct answer for each of these test items. (See Appendix I.) This made available sufficient information in using a particular procedure to compute the test reliability. The particular procedure employed was the Analysis of Variance to Estimate Reliability of Measurements as described by Winer (1971:283-288). A computed reliability coefficient of .92 was found for this test. The result was equivalent to the application of the Spearman-Brown prophecy formula.

Achievement Test Part II consisted of eight question items, but was scored on the basis of a total score of fourteen for each student. Each of the classified testing points, awarded the score of one or zero, was assumed to have the same degree of difficulty. Consequently, the K-R21 formula was chosen to compute the reliability for this part of the test. A Kuder-Richardson Formula 21 reliability coefficient of .82 was found for the test.

Skill Test

The purpose of the Skill Test is twofold: (1) to investigate the students' skill in the use of the fundamental process before studying the unit, and (2) to provide data in assessing the unit contribution to the second objective of junior high school mathematics in Alberta, namely, to develop skills in the use of the fundamental processes.

Skill in the use of the fundamental processes is defined as skill in the arithmetic computation of addition, subtraction, multiplication, and division. The standardized test for Arithmetic Computation in the series Comprehensive Tests of Basic Skills Form Q Level 4 for Grades 8, 9, 10, 11 and 12 put out by the California Test Bureau was used to collect data for the two described purposes.

Arithmetic Computation, Test 6 of the ten basic skill tests in the series, consists of 48 multiple-choice items equally distributed among the four fundamental operations: addition, subtraction, multiplication, and division. The test is arranged with four items in addition appearing in the first column, four items in subtraction in the second column, then four items each for multiplication and division in the third and fourth columns. The arrangement of the four fundamental operations in columns means that each student, working across the columns, is tested in all four operations, regardless of his speed, without having to do a separately timed unit for each operation. The time limit for the test is 36 minutes.

The scoring procedure is straightforward, counting the number of right responses for each of the four operations and the total for the test. Each of the four operations has a score of 12 with a total of 48 for the test. The norms of the test for grade nine are reported in Table 4.6

Table 4.6

Norms of the Standardized Skill Test
Arithmetic Computation

Raw Score in Tested Time of Year			Percentile Rank	Stanine
Sept,Oct,Nov	Dec,Jan,Feb	Mar,Apr,May,June		
46-47	47,48	48	96-99	9
44-45	46	47	90-95	8
41-43	43-45	44-46	78-89	7
37-40	38-42	40-43	60-77	6
30-36	32-37	34-39	41-59	5
23-29	25-31	26-33	23-40	4
17-22	18-24	20-25	11-22	3
13-16	14-17	15-19	5-10	2
0-12	15-19	0-14	1- 4	1

Student Reaction Test

The purpose of the test is to collect data or information about the students' reactions to the instructional unit after having studied it. The test was developed by the author with input from the panel of three professors during the discussion of design, format and other aspects of the test. The complete Student Reaction Test is found in Appendix E.

The student reactions are categorized under five variables: (1) Enjoyment of activity "Do the students enjoy performing the activities during the study of the unit?" (2) Interest of the content "Are they interested in the statistical content in the unit?" (3) Difficulty of the subject "Do they feel that the unit in inferential statistics is difficult for them?" (4) The feeling of new knowledge gained "Do they feel that they gain new knowledge from studying the unit?" (5) Usefulness of statistics "Do they perceive the usefulness of statistics as a result of studying the unit?"

A Likert type questionnaire, with responses categorized according to the degree of agreement or disagreement with the statement, was designed with five items for each of the above variables. The questionnaire includes both positive and negative statements in an appropriate proportion. In all, there are twenty-five questionnaire items and space provided for an optional comment at the end of the test.

The student's response to each questionnaire item is assigned a number within the scale of 1, 2, 3, 4 and 5 with the high score in favor of the unit. For instance, if the response is "strongly agree" to the statement "the unit is difficult," that response is weighed "1".

The test reliability was estimated by using a split-half procedure among odd and even item numbers. However, since the item total of twenty-five cannot be split into two equal parts, the formula by Guttman (1945) was employed. A reliability coefficient of .92 was found for this test.

Teacher Reaction Test

The opinions of the teachers who taught the unit provided another source of information for the unit feasibility assessment. For this purpose the Teacher Reaction Test was developed by the author after consideration and discussion with the panel of three professors.

Realizing the fact that only two teachers would be involved and the information collected would not be appropriate to be analyzed statistically, the general format of the test was designed as an essay type question. The teacher could also be asked to clarify her answers in order to provide more information.

Five questions were asked based on the following areas of required information: (1) The teachers' perception of the unit contributions to the five objectives of junior high mathematics in Alberta, (2) the teachers' perception of the instructional suitability of the unit, (3) the teachers' perception of the demands made on the teachers' subject background in teaching the unit, (4) the teachers' perception of the student reactions to the unit, and (5) the teachers' perception of the unit inclusion in the core program. The first question covered the same areas of information as the Achievement Test and the Skill Test, while the fourth one covered the same area of information as the Student Reaction Test.

In this study, the information was collected by asking the teachers to write their impressions on separate pieces of paper. No attempt was made to score the results; the information was simply summarized.

SUMMARY

The test instruments employed in this study have been described in Chapter 4. The Conception Test is the instrument developed for investigating the students' anchoring ideas about selected statistical concepts before and after the unit instruction. Feasibility assessment in this study is measured with repect to three referenced domains: the unit contribution to the Alberta objectives of junior high mathematics, the student reactions to the unit and the teacher reactions to the unit. The Achievement Test was developed and a standardized Skill Test in arithmetic computation selected to provide data to assess the unit's contribution to junior high mathematics. Similarly, the Student Reaction Test and the Teacher Reaction Test were developed to provide information for assessment of the unit in the second and third areas mentioned above.

The design and conduct of the feasibility study will be presented next in Chapter 5.

CHAPTER 5

DESIGN AND CONDUCT OF THE STUDY

INTRODUCTION

The second and third purposes of this study are: to determine the feasibility of the instruction of the unit for grade nine students, and to investigate the students' anchoring ideas about some selected statistical concepts before and after studying the unit. The development of the instructional unit and the test instruments were described in Chapter 3 and Chapter 4 respectively. Chapter 5 presents a description of the design and conduct of the study as it pertains to the unit feasibility assessment and the investigation of the students' statistical anchoring ideas. Population, sample, teaching staff, classroom setting, operational plan for the instruction of the unit, administration of the test instruments, problems investigated, hypotheses tested, and analysis procedures are described in detail.

POPULATION

The study was conducted in two schools, School A and School B, located in the city of Edmonton, Alberta, during the months of March, April, May and June, 1976. For the sake of convenience, from now on School A and School B will be referred to by using the designations A and B respectively.

Population A

School A was located in a school district covering an area of approximately two square miles and including two junior high schools with a total population of about 900 students. School A housed 435 students in a modern, well equipped building. The surrounding area was partly commercial and partly residential. The socio-economic status of the area was middle class.

The population A for this study consisted of 120 grade nine students divided into four classes which made up the entire 1975-76 grade nine population of School A. In this school there were three mathematics teachers.

Population B

School B was located in a school district covering an area of about two square miles, and including approximately 500 junior high students. School B was a combined elementary-junior high school which housed 240 elementary pupils and 130 junior high students in a modern, well equipped building. The surrounding area of School B was residential, the socio-economic status of which was described as low middle class.

Population B for this study consisted of 47 grade nine students divided into two regular classes which made up the entire 1975-76 grade nine population of School B. There were two mathematics teachers at the junior high level of School B.

THE SAMPLES

The two samples employed in this study were not randomly selected from the population described. Instead, they were picked on the basis of School A and School B agreeing to make classtime available. Such nonrandom samples can be good representatives of a target population if they are carefully described. Therefore, information about, and characteristics of, the selected samples are important. The sample selection and description of sample background are presented next.

Sample Selection

Two samples, Sample A and Sample B, were chosen from the above described populations. Sample A consisted of 21 students chosen from a population of 120 grade nine students in School A. Sample B consisted of 27 students chosen from a population of 47 grade nine students in School B. Both the samples were formed by the schools and the teachers into what is called "an option class." By its nature, an option class generally is expected to be structured on the basis of the interests, needs, and abilities of the students. And it is expected to be structured by the individual school. Before choosing to take the option, the students in the samples were informed that they would study a unit in statistics in the option classes.

Practically, the students in Sample A personally selected, according to their own interest, to join the math option class which met twice a week. The students came from three regular classes in which they also studied a regular mathematics course—algebra—three forty-minute periods per week.

The students in Sample B chose to join the math option class from the two available choices of math option or French option, each of which offered three periods per week at the same time. The students in the math option class came from two different regular classes in which they also studied algebra for three periods per week.

Sample Description

Sample A consisted of 21 grade nine students including 10 boys and 11 girls. The average age of the students in the sample, up to April 1976, was 178.10 months and the standard deviation was 4.48 months. I.Q.'s of the students in the sample were measured by using the Lorge Thorndike Intelligence Test in November 1974. The mean I.Q. was 107.43, and the standard deviation was 15.51. Three students had I.Q. scores lower than 90, and eleven students had I.Q. scores higher than 110. Among these eleven, four students had reached 120 or more. The average grade for mathematics in the previous term was 71.19%, and the standard deviation was 18.57%. Eleven students were over the average grade and five students were equal to or lower than the grade of 50%.

The purpose of this sample description is to provide as much relevant information as possible about the selected sample. Because skill in the use of fundamental arithmetic operations is important for studying a unit in statistics, especially where the students have to do computations themselves, it is worthwhile to include a description of the arithmetic computational skill of the sample before studying the unit.

A standardized test measuring arithmetic computational skills was given in February 1976. It had a mean of 33.76 and a standard deviation of 11.56 from a total score of 48. According to the norms of the standardized test, with respect to the testing time as mentioned, the mean of 33.76 is at the percentile rank of 44 which is in the lower half of a stanine of five, or a middle group.

Sample B consisted of 27 grade nine students including 21 boys and 6 girls. Up to April 1976, the average age of the sample was 178.63 months, and the standard deviation was 5.52 months. The sample's I.Q. scores were measured by using the Lorge Thorndike Intelligence Test during the time the students in the sample studied in grade eight, before Christmas 1974. The mean I.Q. was 101.11, and the standard deviation was 12.01. Among 27 students, there were 6 students who had I.Q. scores lower than 90, and there were also 6 students above the I.Q. score of 110. The average grade for mathematics in the previous term was 52.59%, and the standard deviation was 19.68%. Twelve students had mathematics grades above the average, half of which got a mathematics grade of 70% or more. Among the group lower than the average grade, six students had a mathematics grade of 50% and six students got a lower grade than 40%. The arithmetic computational skill as measured by a standardized test in the middle of March 1976, had a mean of 35.26 from a total score of 48, and a standard deviation of 7.84. With respect to the norms of the standardized test, the mean of 35.26 locates about the percentile rank of 50 which is about the center of a stanine of five, or a middle group.

Sample A and B are summarized in Table 5.1. More detail can be found in Appendix G. In Sample A, about two thirds of the students were considered to be in a middle or higher I.Q. group, while about

Table 5.1

Summary of Sample Background

	Sample A		Sample B	
Charact. Meas.	$\overline{x}$	SD	$\overline{x}$	SD
I.Q.	107.43	15.51	101.11	12.01
Math Grade (%)	71.19	18.57	52.59	19.68
Age (months)	178.10	4.48	178.63	5.52
Arith. Comp. (of 48)	33.76	11.56	35.26	7.84

one third of those belonged to a lower group. Sample B had just about the opposite characteristics of Sample A, that is, about one third were considered to be an above average group and two thirds belonged to the middle and lower groups.

Sample A had a considerably higher previous mathematics grade average—71.19%—than Sample B—52.59%. The average ages of the two samples were very close. Sample B had higher scores on arithmetic computational skill than Sample A, however, Sample A took the standardized test in arithmetic computational skill about a month before Sample B.

TEACHING STAFF

The teaching staff consisted of two teachers, called Teacher A and Teacher B, and the author who acted as an observer, a resource person, and a helper in checking the students' attendance. Both teachers were female and were chosen on a voluntary basis of one out of three mathematics teachers in School A and one out of two in School B.

Approximately four hours of inservice training was given separately to each of the two teachers by the author about two weeks before starting the instruction of the unit. The emphasis of the inservice training was concerned with the content and the instructional approach in the unit. Two sets of the constructed lessons, a teacher copy and a student copy, were used to illustrate and discuss the intent of the course during the inservice period.

Teacher A is an experienced teacher, who has taught mathematics at both the levels of senior high and junior high school for a number of years. She has two university degrees, a B.Ed. and a B.A., along with eight university mathematics courses.

Teacher B has six years' experience, all in School B. She holds the university degree of B.Ed., majoring in mathematics. She has studied six university mathematics courses, including algebra, geometry, calculus, and one introductory statistics course.

CLASSROOM SETTING

It is not the purpose of this study to make a direct comparison of the results between the two selected samples. However, an attempt is made to point out that different results of the study may be due to a difference in settings and environment. In addition, the more information that is available surrounding the selected samples and the conduct of the study, the better the inferences of the study can be interpreted in terms of the target populations. Therefore, classroom facilities and equipment, and the instructional setting are described.

Classroom Facilities and Equipment

Classroom A is a large open rectangular area, about 25 feet in width and 40 in length. It is modern and very well equipped. There are chalkboards covering three sides, including one long side. On these three sides there are also three bulletin boards attached to the three chalkboards in the corners of the room. There is a movable graph board on a stand located in the remaining corner of the room. On one end wall there is a retractable overhead projector screen. There are two doors, one at each end of a long wall, in between which is a small room for housing materials for this classroom. There are two sets of student desks, regular individual desks for when the class is organized as a whole, and a six sided desk, which can also be split in half, for large or small group work.

Classroom B is quite small in comparison to classroom A. It is about 17 feet wide and 25 feet long. On one short wall, which is considered the front of the classroom, there are a chalkboard and a retractable projector screen; on the back wall there is a bulletin board. The long walls form the sides of the room. On the left are the windows with pull blinds and a long narrow counter under which are several shelves for keeping materials; on the right are a spare chalk board and the only door. There are only regular individual type student desks. However, they can be moved together for group work when needed.

The prepared materials covering the unit of study were given to the teachers. Each teacher had both sets of lessons, a teacher copy and a student copy, for her instructional planning. Each student was given a typed copy of the lessons, but not the whole set at the beginning, only the part they were supposed to study in class at the time. The materials for doing experiments, such as a number of boxes each of which included the numbered square railroad paper, were prepared by the author and given to the teachers on the day having that particular lesson. Appropriately marked graph paper, and various prepared recording sheets were available to the students.

Besides the above mentioned materials, the students had their own regular learning materials such as pencils, pens, rulers, and so on. In addition the students in Sample A also had boxes of magic colors for painting graphical figures in the examples or exercises during the study of the unit. Some of their work was displayed on the bulletin boards in the classroom.

Instructional Setting

Some lessons called for the instruction of the class as a whole, while other lessons called for small group instruction for doing experiments. When instructing the class as a whole, the teacher explained or illustrated the examples chosen to clarify the concept being taught; at the same time the students were instructed to individually do their part, such as to complete the examples or answer key questions appearing on the student copy of the lesson. The teacher walked around to help individual students when needed.

During those lessons which required experiments, the students were instructed to form groups of two of their own choice to perform the activities. One student performed the experiment, while the other recorded the results. After that, the students individually did the activities involved in recording data from the experiment such as drawing graphs, answering leading questions on the specific concepts demonstrated by the experiments, and making notes on the conclusion.

In case of the student's absence from a regular class, both teachers assigned extra work for the student to do at home or some other time in school.

OPERATIONAL PLAN FOR THE INSTRUCTION

The unit of instruction was planned to include approximately twenty forty-minute periods of teaching time. Table 5.2 and Table 5.3 illustrate the actual operational plans for the unit of instruction in the two schools, described briefly day by day.

School A used eighteen forty-minute periods and School B used twenty-two forty-minute periods as their actual instructional time. However, School A started teaching on March 11 which was almost a month before School B started on April 7, 1976. Both schools finished teaching on the same day—June 14. The class schedules for teaching the unit were different—two periods per week for School A, and three periods per week for School B.

In School A, the first two periods were used for up-dating the students' background about graphs and some statistical terms. The next thirteen periods were used in teaching Lessons 1 to 9, while only the last three periods were used in covering the content of Lessons 10-15. The instructional time for the last portion was very rushed due to the lack of available instructional time. However, Teacher A got some help from the author and one professor in modifying the lessons to suit the time available and reviewing Lessons 12-15 for the students.

For School B, the first lesson from the unit was used for the first period of instruction and the schedule went on regularly in the first nineteen periods. The last three periods were used to cover the content of Lessons 13-15 which would normally have taken more instructional time. The author and one professor helped in

Table 5.2

Day-by-Day Instructional Plan for School A

Period Number	Date	Lesson Number	Remarks
1	March 11	Introduction to various graphs	e.g. bar graphs, chart, circle graphs, pictorial graphs.
2	18	Introductory definition of terminology	e.g. data, frequency, and other terms appearing in Lesson 1.
3	19	1	
4	April 8	2	
5	9	2	Some students did activities for Lesson 2, but some had to complete their preparatory graph work before going on to Lesson 2 activities.
6	15	Introd. to L.3	Developed by the teacher.
7	22	3	
8	23	3	
9	29	4	
10	30	4,5	
11	May 6	5 (cont.)	
12	7	6	
13	14	6,7	
14	20	8	
15	21	8,9	
16	27	10	Lesson 11 and activity for L.10 were skipped. Lesson 12 was given to the students to study themselves.
17	June 3	13,14	The author and one professor helped in setting the experiments. The students were instructed to complete drawing graphs at home.
18	14	15	The concepts for L.13,14 were explained, partly by the author. Only experiment part of L.15 was done. The author had about 10 minutes extra time to review L.12 to 15 for the students.

Table 5.3

Day-by-Day Instructional Plan for School B

Period Number	Date		Lesson Number	Remarks
1	April	7	1	
2		12	2	Reviewed Lesson 1 before starting.
3		13	2 (cont.)	Needed more time to draw graph.
4		19	3	
5		26	4	
6		27	4 (cont.)	
7		30	5	Some students still had difficulty in drawing graphs.
8	May	4	5 (cont.)	
9		5	6	
10		7	6 (cont.)	
11		11	7	
12		12	7 (cont.)	
13		14	Quiz	Questions developed by the teacher.
14		18	8	
15		19	Report quiz results	Reexplained Lesson 7.
16		21	9	
17		25	10	
18		26	11	
19	June	1	12	Also explained Lesson 11.
20		2	13,14	The author and one professor helped in setting the experiments for Lessons 13,14.
21		9	13,14	Drew graphs from experiment data. The author helped explain the intended concepts.
22		14	15	Completed only the experiment part of lesson. Review sheets developed by the teacher were given to the students to study by themselves.

teaching some sections, similar to what was done for School A.

TEST INSTRUMENT ADMINISTRATION

Pretest Administration

For the purposes of this study, it was necessary to administer only the Conception Test and the Skill Test as pretests. The Skill Test was administered by the author with the help of the teachers from both schools—late in February 1976 for School A, and in the middle of March 1976 for School B. Three students from School B who missed the first test administration were given the test a week later.

The Conception Test for School A was administered between the middle of February and the tenth of March, 1976. Due to the limited time provided by the school to administer the test during the above period, seventeen students were randomly selected out of the total of twenty-one. They were interviewed and their answers were taped.

The Conception Test for School B was administered between the last week of February and the sixth of April, 1976. Due to the limited amount of time for administering the test during that period, twenty-five students out of the total twenty-seven were randomly selected to take the test. Furthermore, the testing procedure had to be modified. Instead of interviewing and taping one by one, the students were formed into groups of five, one group taking the test at each time. To prevent the students from influencing each other, the five students were instructed to sit apart in a regular classroom and to independently write the answers on the paper provided. They were also instructed to write their answers as if being interviewed, and to individually ask the interviewer for clarification of the questions or the problems appearing on the test when needed. During the testing time, the interviewer also walked around to check the answers of each of the five students and asked them to clarify some points in their answers if necessary.

Although it was necessary to change the format for administering the Conception pretest from the individual interview testing procedure originally designed to the group testing procedure just described, care was taken in designing the change in format to ensure that the results would be the same—that is, that the statistical anchoring ideas of as many students from the selected sample as possible would be investigated.

Post-test Administration

Although only the Conception Test and the Skill Test were administered as pretests, all test instruments employed in this study were administered as post-tests. The administration of the post-tests for both schools was completed within ten days after the last lesson of the unit was taught. The order for administering the tests was planned in the sequence of: (1) the Student Reaction Test, (2) the Skill Test, (3) the Achievement Test, and (4) the Conception Test. The purpose in setting the administration of the tests in such a sequence was to minimize the influence the taking of one test could have on another. In order to catch the students' reactions to the course while they were still fresh and not influenced by, for instance, any difficulty with the Skill Test, the Student Reaction Test was administered first. Similarly, the Skill Test was administered before the Achievement Test so that it would test the results of studying the course on the students' computational skills before, for instance, those skills could be influenced by problems on the Achievement Test involving computations.

The Conception Test was intended to test the students' statistical anchoring ideas, which were in the form of generalities and abstractions, as they had been influenced by the instruction of the unit. It was also intended to partly measure the retention of those new anchoring ideas, not the immediate memorized facts from the unit of instruction. Therefore, the administration of the Conception Test was assigned to last place in the sequence of post-tests. Both schools managed to follow that assigned order for administering the tests. The Teacher Reaction Test was given to both teachers after the completion of the unit of instruction, and in order to have them express their opinions in writing, there was no time limit.

For School A, only one student missed the Skill Test. For School B, three students missed the Achievement Test and two students missed the Skill Test. Due to the limitations of time, the students who missed tests were cut off from the number of students in the samples for those particular tests.

The amount of testing time was arranged so that the students could use as much time as they wanted to complete the tests, except for the Skill Test which had to be completed within 36 minutes as set by the standardized test. The Conception Test was administered to a whole group by instructing the students to individually write the answers in the provided spaces after the questions in the test. They were also instructed to answer as if they were being interviewed. The students completed the Conception Test within one forty-minute period.

The change in the administration format of the Conception Test from testing individuals or a small group of students to testing the whole class at the same time was necessary because there was only one period available from each of the two schools to administer the test. It was important to ensure that all students who took the pretest had a chance to take the post-test. However, the testing procedures, such as informing students to answer as if being interviewed, were similar to those employed in a small group. Furthermore, because the students already had had the experience of taking the pretest, they were familiar with the test and procedures.

PROBLEMS TO BE ANSWERED AND HYPOTHESES TO BE TESTED

The problems and hypotheses are established with respect to the three purposes of the study to determine the constructibility of the unit, to assess the feasibility of using the unit, and to investigate the students' anchoring ideas on some statistical concepts. The constructibility is defined as the unit appropriateness. The feasibility is defined with respect to three referenced areas: the unit contribution to the five mathematics objectives, student reactions, and teacher reactions. Both appropriateness and feasibility were defined in Chapter 1.

Unit Constructibility

Problem: Can a unit in basic inferential statistics be constructed in such a way that it is appropriate for grade nine students and is also appropriate for the five objectives of junior high school mathematics of Alberta as listed below?

1. To develop an understanding of mathematical concepts and appreciation of mathematical structure.
2. To develop skill in the use of the fundamental process.
3. To develop systematic methods of analyzing problems and of presenting their solutions.
4. To develop habits of precise thought and expression.
5. To develop an understanding of the significance and appreciation of mathematics in the modern world.

Feasibility Assessment

1. The unit contribution to the objectives of Alberta junior high mathematics.

1.1 To develop the understanding of mathematical concepts and structures.

Problem: Do the students who study the unit in inferential statistics develop an understanding of statistical concepts and structures, as measured by the Achievement Test Part I, which reaches or exceeds the criterion referenced measurement of the test?

1.1a For the students in School A.

1.1b For the students in School B.

1.1c For the students in both School A and School B.

1.2 To develop skill in the use of the fundamental process.

Null Hypothesis: There are no differences between the students' skills in the use of the fundamental operations (addition, subtraction, multiplication, division, and the four operations combined), as measured by the standardized test of arithmetic computation, before and after the instruction of the unit.

1.2a For the students in School A.

1.2b For the students in School B.

1.2c For the students in both schools.

1.3 To develop systematic methods of analyzing problems and of presenting their solution.

Problem: Do the students who study the unit in inferential statistics develop methods of problem solving, as measured by the Achievement Test Part II, which reach or exceed the criterion referenced measurement of the test?

1.3a For the students in School A.

1.3b For the students in School B.

1.3c For the students in both School A and School B.

1.4 To develop habits of precise thought and expression.

Problem: Do the students who study the unit in inferential statistics express some observable behavior which indicates they are developing habits of precise thought and expression?

1.5 To develop an understanding of the application of mathematics in the modern world.

Problem: Do the students who study the unit in inferential statistics develop an understanding of the application of statistics, as measured by the subset of the Achievement Test Part I, which reaches or exceeds the criterion referenced measurement of the test?

1.5a For the students in School A.

1.5b For the students in School B.

1.5c For the students in both School A and School B.

2. Feasibility assessment with respect to the student reactions.

2.1 The problems to be answered:

How do the students, after having studied the unit in basic inferential statistics, react to the unit with respect to each of the five variables: (1) enjoyment of activities, (2) difficulty of the subject, (3) interest of the content, (4) belief of new knowledge gained, (5) usefulness of statistics; and all five variables combined as measured by the Student Reaction Test?

2.1a For the students in School A.

2.1b For the students in School B.

2.1c For the students in both schools.

2.2 The null hypothesis to be tested in relation to the significance of the students' reactions to the unit of instruction to the feasibility assessment:

There are no differences in the numbers of the responses between "favorable" and "unfavorable" on the student reactions to the unit with respect to each of the five variables: (1) enjoyment of activities, (2) difficulty of the subject, (3) interest of the content, (4) belief of new knowledge gained, (5) usefulness of statistics; and all five variables combined as measured by the Student Reaction Test.

2.2a For the students in School A.

2.2b For the students in School B.

2.2c For the students in both schools.

3. Feasibility assessment with respect to the teacher reactions.

Problem: What opinions do the teachers who teach the unit in basic inferential statistics have about each of the following five considerations: (1) perception of the unit contribution to the objectives of Alberta junior high school mathematics, (2) perception of the instructional suitability of the unit, (3) perception of the demands made on the teacher's subject background, (4) perception of the student reactions to the unit, and (5) perception of the unit's being included in a regular mathematics program—as measured by the Teacher Reaction Test?

3a For the teacher in School A.

3b For the teacher in School B.

The Investigation of the Students' Anchoring Ideas

1. Problem: What are the students' anchoring ideas about each of the following selected statistical concepts: (1) measures of central tendency, (2) the transformation of a mean when a constant is being added to each datum, (3) the transformation of a mean when each datum is being multiplied by a constant, (4) a normal distribution, (5) the use of a sample mean to estimate a population mean, (6) variability of scores, (7) the tranformation of a variance when a constant is being added to each datum, and (8) the use of a mean from a different size sample to estimate a population mean—as measured by the Conception Test before and after the instruction of the unit?

   1a For the students in School A.

   1b For the students in School B.

   1c For the students in both schools.

2. Problem: How do students in both School A and School B change their anchoring ideas between the pretest and post-test, as measured by the Conception Test, on the following statistical concepts: (1) measures of central tendency, (2) the transformation of a mean when a constant is being added to each datum, (3) the transformation of a mean when each datum is being multiplied by a constant, (4) a

normal distribution, (5) the use of a sample mean to estimate a population mean, (6) variability of scores, (7) the transformation of a variance when a constant is being added to each datum, and (8) the use of a mean from a different size sample to estimate a population mean?

ANALYSIS PROCEDURES

This study by its nature is analytic and descriptive. The data for some of the problems are measured using criterion-referenced instruments. For other problems, simple descriptive statistics are used to analyze data. However, some inferential statistical techniques are used to test the stated hypotheses. Following are the descriptions of the procedures employed in this study to analyze the problems mentioned in the previous section.

Analysis for Unit Constructibility

The problem of assessing the unit's constructibility is descriptively analyzed by focusing on its appropriateness for grade nine students and for the five objectives of the junior high school mathematics program of studies. In particular, the basis for assessing the unit's constructibility comprises the design and construction procedures as described in Chapter 3, the statistical content included, and the instructional approach selected. In addition, the results of the Student Reaction Test on the variable "difficulty" are used to assess appropriateness for grade nine students. Finally, some results of the feasibility assessment are used to assess the unit's appropriateness for the five objectives of the junior high school mathematics program.

Analysis for the Feasibility Assessment

The Unit's Contribution to the Objectives of Mathematics

The feasibility assessment of the unit's contribution to the objectives of the Alberta junior high school mathematics program is made using the following criterion-referenced measurements: Achievement Test Part I, Achievement Test Part II, and the subset of Achievement Test Part I—items classified under the category of "application" as described in Chapter 4. This relates to Problem 1.1, Problem 1.3, and Problem 1.5.

The setting of criterion references is done in two steps. In the first step a percentage of correct responses is set as a criterion for a test. In the second step the proportion of the students who reach or exceed this criterion on the test is set.

The first step in criterion referencing was described in Chapter 4. For the second step in criterion referencing, care must be taken in considering the nature of the students who study the unit and take the test. In particular, for this study, the nature of the students was defined in terms of I.Q. scores and partially in terms of their previous grades in mathematics, as illustrated in Appendix G. As a consequence, in the second step, the criterion reference was set as the proportion of the students who have I.Q. scores equal to or greater than one hundred.

The criterion references for feasibility of the unit contribution according to Problem 1.1, Problem 1.3 and Problem 1.5 are symbolized as (c:n) where "c" represents the criterion referenced measurement of the test related to the first step in criterion referencing, and "n" represents the criterion referenced proportion of the students as related to the second step in criterion referencing. In this study, Table 5.4 illustrates the criterion references employed for the feasibility assessment.

To test the null hypothesis 1.2, including all sub-hypotheses, about the development of skill in the use of the fundamental operations, the t-test procedure dealing with the difference between two means on correlated observations is employed. The assumption of a normal population was checked by plotting the graphs of the testing results for each and both schools. The significant level for the hypothesis testing was set at .05.

Finally, Problem 1.4, relating to the development of habits of precise thought and expression, is answered by descriptively analyzing the evidence from classroom observations during the instruction of the unit and some results of the Conception Test. The analysis is focused on the students' observable behavior which indicated the development of habits of precise thought and expression about statistics as a result of studying the unit.

Student Reaction Analysis

The feasibility assessment with respect to the student reactions to the unit is analyzed using both a descriptive procedure and a statistical technique.

To test the null hypothesis 2.2, including all sub-hypotheses, about the favorable and unfavorable student reactions to the unit on

Table 5.4

The Criterion Referenced Feasibility Assessment

Problems Tested	Instrument Employed	School	(c:n)
1.1	Achievement Part I	A	(27/45:14/21)
		B	(27/45:13/24)
		A & B	(27/45:27/45)
1.3	Achievement Part II	A	(7/14:14/21)
		B	(7/14:13/24)
		A & B	(7/14:27/45)
1.5	Achievement Part I (Subtest)	A	(9/18:14/21)
		B	(9/18:13/24)
		A & B	(9/18:27/45)

the specified variables, the chi square goodness of fit testing procedure is employed. The significant level of .05 is set for the hypothesis testing.

"Favorable" and "Unfavorable" need clarification. For each variable in the Student Reaction Test, there are five scales of responses: 1, 2, 3, 4, 5. "Favorable" is the number of responses on the specified variable rating in the scales of 4 and 5 and also half the number of responses in the scale of 3. The rest of the responses on the same specified variable are counted as "Unfavorable."

For Problem 2.1, about the student reactions, a descriptive procedure is used. The mean and standard deviations are computed for each variable and for the total instrument. In addition, the number of responses as well as the percent are illustrated in each of the five scales of answers for every variable and for the total instrument.

Teacher Reaction Analysis

For Problem 3, about the teacher reactions to the unit, a descriptive procedure is used. The opinions expressed by each teacher on a specified question are descriptively summarized.

Analysis for the Investigation of Anchoring Ideas

Two stated problems, as mentioned in the previous sections, are to be answered and analyzed; and for both problems, a descriptive procedure is employed.

For Problem 1, the investigation of the students' statistical anchoring ideas before and after the instruction of the unit, the results of both the pretest and the post-test are descriptively analyzed. For each of the selected statistical concepts in the test questions, the student answers are listed and ranked in order according to the quality of the answers. The frequency of responses of each of these different answers is recorded and reported in percent and also in total number. More detail can be found in Chapter 4 concerning the scoring procedure for the Conception Test.

For Problem 2, the investigation of changes in student statistical concepts as indicated by the results of the pretest and the post-test, a descriptive analysis is employed. The analysis that is used is similar to that used for Problem 1. For each of the selected statistical concepts the student answers on both the pretest and the post-test are combined in the same list of ranked answers. From this list of ranked answers, some of the answers are categorized as "satisfactory," or "s"; and some are categorized as "unsatisfactory," or "u". Furthermore, the satisfactory level is subcategorized as s^+, s, s^-. The categorized levels are defined in terms of the quality of the answers for each of the selected statistical concepts, and will be illustrated as a part of the results for Problem 2 in Chapter 6. With respect to the above described categorization, the format representing changes in the students' anchoring ideas between the pretest and the post-test is designed in a matrix form. Then the results are reported in both percent and also in total number of students according to the format representing changes in the designed matrix form. Full detail of the analysis procedure for Problem 2 is presented in Chapter 6, The Results of the Study.

SUMMARY

Chapter 5 described the design and actual conduct of bringing the developed materials, the instructional unit and the test instruments into operation in order to gather data or information to answer the problems or to test the hypotheses set for the unit feasibility assessment and the investigation of the students' anchoring ideas. Chapter 5 also described the design of analysis procedures for the collected data in preparation for the actual analysis of results described in the next chapter.

Specifically, the descriptions of population, sample, teaching staff, classroom setting, operational plan for the instruction of the unit, test instrument administration, problems to be answered and hypotheses to be tested, and the analysis procedures employed were presented in Chapter 5. Chapter 6 carries on the task of presenting the results of the study.

CHAPTER 6

RESULTS OF THE STUDY

The problem in this study has been stated in its threefold purpose—the assessment of: (1) the constructibility of the unit, (2) the feasibility of using the unit to teach grade nine students, and (3) the investigation of the students' anchoring ideas on some statistical concepts before and after the unit instruction. Chapter 6 presents the results of the study relating to the above three major aspects. The problems to be answered and the hypotheses to be tested are restated. The analysis procedures for those problems and hypotheses are described followed by the presentation of the results. Some direct conclusions regarding each of the results presented are also made.

THE UNIT CONSTRUCTIBILITY

Problem Restated

> Can a unit in basic inferential statistics be constructed in such a way that it is appropriate to grade nine students and is also appropriate to the five objectives of junior high school mathematics of the Province of Alberta listed below?
>
> (1) To develop an understanding of the mathematical concepts and appreciation of mathematical structure.
> (2) To develop skill in the use of the fundamental processes.
> (3) To develop systematic methods of analyzing problems and of presenting their solutions.
> (4) To develop habits of precise thought and expression.
> (5) To develop an understanding of the significance and appreciation of mathematics in the modern world.

Analysis Procedure

The unit constructibility is descriptively analyzed with respect to the unit construction as described in Chapter 3 and also incorporated with the feasibility assessment presented in the next section. The analysis is focused on the appropriateness of the unit for grade nine students and the five objectives of mathematics mentioned above.

The first part of the analysis of the unit appropriateness for grade nine students is focused on the justification of the content selection, the prerequisites defined, the instructional approach selected, and the main instructional strategy designed. The second part of the analysis brings in the empirical evidence that grade nine students can adequately learn the statistical concepts with respect to the specific objectives of the unit described in Chapter 3, and also the evidence of the student reactions to the unit on the variable "difficulty of the subject."

For the analysis of the unit appropriateness for the five objectives of mathematics, the first part is the justification of the assertion that the unit fulfills the five objectives, and the second part brings in the empirical evidence to support that justification.

Results

A. The Unit Appropriateness for Grade Nine Students

1. From the unit construction as illustrated in Chapter 3.

1.1 The Content Selection: The statistical content included in the unit was selected on the basis of the content recommended by various reports such as the Cambridge Conference and the Undergraduate

Committee on Statistics. Moreover, the content was selected with respect to the consideration of its psychological, social, and mathematical merit.

1.2 The Prerequisites Defined: The prerequisites of the unit were chosen on the grounds that general grade nine students should already have attained certain skills, such as the ability to construct and read various graphs, and skills in the fundamental arithmetic operations. Moreover, probability, which is generally believed to be the major prerequisite of inferential statistics, was not treated as a rigid prerequisite for this unit. Instead, it was designed to be involved in a minimal and intuitive way such as the average grade nine student should have had in his intuitive experience or can intuitively grasp from the teacher during the course of instruction.

1.3 The Instructional Approach Selected: The instructional approach selected for the unit was generally intuitive. This approach was recommended by various reports advocating that it is appropriate for young children like grade nine students.

1.4 The Instructional Strategy Designed: The main instructional strategy employed in this unit was designed to suit the nature of young students. Statistical concepts were illustrated by using examples along with graphical figures and also by doing experiments when appropriate. Moreover, for motivating purposes, the students' personal data were used in studying the unit.

2. Incorporation with some results of the feasibility assessment.

2.1 The result of the feasibility assessment of the unit contribution to the first objective of junior high school mathematics, illustrated in Table 6.1 (page 170), also indicates the unit appropriateness for the students as to whether they can adequately learn the statistical concepts from the unit. The table illustrates the results, with respect to the set criterion referenced measurement, as follows:

2.11 80% of the students adequately learned the concept of "population and sample."

2.12 73.37% adequately learned the concept of "sampling with and without replacement."

2.13 75.56% adequately learned the concept of "frequency distribution."

2.14 84.44% adequately learned the concept of "measures of central tendency."

2.15 60% adequately learned the concept of "measures of variability."

2.16 68.89% adequately learned the concept of "score transformation."

2.17 53.33% adequately learned the concept of "a normal distribution."

2.18 55.56% adequately learned the concept of "central limit theorem."

2.19 95.56% adequately learned the concept of "statistical estimation."

2.20 60% of the students adequately learned the statistical concepts from the unit.

2.2 The result of the unit feasibility assessment with respect to the student reactions, in particular on the variable "difficulty of the subject," as illustrated in Table 6.5 (page 182), also indicates the unit appropriateness from the students' point of view.

The results indicate that: 4.58% of the students strongly agreed, 21.67% agreed, 16.25% undecided, 52.92% disagreed, and 4.58% strongly disagreed with the statement "The unit is difficult."

B. The Unit Appropriateness for the Five Objectives of Mathematics

1. The Justification of the Assertion that the Unit Fulfills the Five Objectives.

The general procedure employed in constructing the unit as described in Chapter 3 started with the analysis of two main variables—the content and instruction. It was the function of these two variables, as they were designed to be included in the unit, to serve each of the five objectives.

1.1 The unit in inferential statistics, like other mathematical units, provided various statistical concepts and structures, as described in Chapter 3 on the content analysis, to serve the first objective of junior high school mathematics. The intuitive approach, considered as the appropriate instructional procedure, was selected for the unit in order to help the students to develop an understanding of those intended concepts.

1.2 The unit provided various materials suitable for the practice of skill in the fundamental arithmetic operations, for example, the lessons on measures of central tendency, measures of variability, and score transformation. However, this unit was constructed in such a way that the instruction compromised between the practice of skill in the fundamental process and emphasis of the intended statistical concepts. The students had to do the computation, although some illustrated examples in the unit were constructed using a small set of data with small numbers in order to save time for manipulating.

1.3 The unit was constructed in such a way that it provided the problem solving methods involved in the statistical descriptive procedure, the statistical inferential procedure, and the general statistical procedure to serve the purpose of the third objective of junior high school mathematics. Various concepts included in those procedures were taught and illustrated such that the students developed the ideas of those intended problem solving methods.

1.4 The unit was constructed in such a way that it provided means for developing habits of precise thought and expression. Statistical terms, formulas, and definitions were taught and illustrated in order to help the students develop precise ideas about those intended concepts by thinking them through and expressing them. In addition, symbols of statistical terms, formulas, and definitions were introduced and intentionally used throughout the process of the unit instruction.

1.5 Statistics, by its nature, is applied mathematics. This unit was constructed in such a way that the students developed an understanding of the concepts and the applications of those concepts.

The students' personal data included in the unit, besides motivating the students to become involved in the study, also served the purpose of applying the statistical concepts to those personal data. However, the unit instruction was designed to compromise between simple created data to illustrate the intended concepts and real data for applying or illustrating those concepts because the real data tended to have the uncontrolled disadvantage of involving complex manipulation and computation that consumed too much instructional time and bored some students.

2. Incorporation with some results of the feasibility assessment.

The unit appropriateness for the five objectives of mathematics was analyzed with respect to the unit construction. The unit was designed and constructed such that it was appropriately expected to serve each of the five objectives as already described above.

The results of the feasibility assessment of the unit's contribution to the five objectives of junior high mathematics are incorporated here as further evidence of the unit's appropriateness. These results and conclusions are described next.

FEASIBILITY ASSESSMENT

1. The Unit Contribution to the Objectives of Alberta Junior High Mathematics

1.1 Problem Restated

Do the students who study the unit in basic inferential statistics develop the understanding of statistical concepts and structures, as measured by the Achievement Test Part I, reaching or exceeding the criterion referenced measurement of the test?

1.1a For the students in School A
1.1b For the students in School B
1.1c For the students in both schools.

Analysis Procedure

The problem is analyzed with respect to the two criterion referenced dimensions: the criterion referenced measurement of the test, and the criterion referenced number of students who achieve the criterion referenced measurement. For the second dimension, the criterion referenced number of students is set as follows: 14 out of 21 students in School A, 13 out of 24 students in School B, and 27 out of 45 students for both schools. (The Criterion Referenced Feasibility Assessment is illustrated in Table 5.4, page 158.)

Results

Table 6.1 illustrates the results of assessing problems 1.1a, 1.1b, and 1.1c. The table presents the criterion scores for the test with respect to each of the seven specific objectives of the unit, and also the criterion scores for the whole Achievement Test Part I. The numbers and percentages of the students, for each and both schools, who achieved the criterion scores for each item and the whole test are also reported. The means and standard deviations of the students' scores for each and both schools are given for the purpose of describing the students' achievement.

Table 6.1

The Analysis of the Results of Achievement Test Part I

The Concepts to be Tested	Criterion Referenced Scores	Number (%) of Students Achieving		
		School A	School B	A & B
1. Population and sample	2/4	15 (71.43)	21 (87.50)	36 (80.00)
2. Sampling with and without replacement	2/3	15 (71.43)	18 (75.00)	33 (73.33)
3.				
3.1 Frequency distribution	7/10	17 (80.95)	17 (70.83)	34 (75.56)
3.2 Measures of central tendency	4/7	17 (80.95)	21 (87.50)	38 (84.44)
3.3 Measures of variability	2/3	13 (61.90)	14 (58.33)	27 (60.00)
4. Score transformation	3/6	13 (61.90)	18 (75.00)	31 (68.89)
5. A normal distribution	1/2	15 (71.43)	9 (37.50)	24 (53.33)
6. Central limit theorem	3/5	13 (61.90)	12 (50.00)	25 (55.56)
7. Estimation	3/5	19 (90.48)	24 (100.00)	43 (95.56)
Total: Achievement Test Part I	27/45	14 (66.67)	13 (54.17)	27 (60.00)
$\bar{X}$ of the students' scores	-	30.57 (67.93)	29.71 (66.02)	30.11 (66.91)
SD of the students' scores	-	10.29	8.01	9.05

Conclusion

Fourteen out of 21 students in School A, 13 out of 24 students in School B, and 27 out of 45 students in both schools achieved the criterion referenced scores. Therefore, the students in School A, School B and both schools adequately developed an understanding of statistical concepts and structures from the unit.

1.2 Null Hypothesis Restated

> There are no differences between the students' skills in the use of the fundamental operations (addition, subtraction, multiplication, division, and the four operations combined), as measured by the standardized test (Comprehensive Tests of Basic Skills, Form Q4—Arithmetic: Test 6—Arithmetic Computation), before and after the instruction of the unit.
>
> 1.2a For the students in School A
> 1.2b For the students in School B
> 1.2c For the students in both schools.

Analysis Procedure

The hypothesis was tested by employing the t-test procedure for testing the difference between two means of correlated observations. The significant level of .05 was set in the hypothesis testing.

Results

Table 6.2, Table 6.3, and Table 6.4 illustrate the results of testing the null sub-hypotheses 1.2a, 1.2b, and 1.2c. Each of these tables gives the test of fundamental operation tested, the mean ($\bar{d}$) and the standard deviation (sd) of the students' difference scores between pre and post-tests, degree of freedom (df), the value of observed t score, and the probability (p) of the observed t value.

Table 6.2

The Analysis of the Skill Test Results for School A

Operation Tested	$\bar{d}$	sd	df	t	p
Addition	.15	1.50	19	.45	$.6<p<.7$
Subtraction	.80	1.51	19	2.37	$p<.05$*
Multiplication	1.25	2.31	19	2.42	$p<.05$*
Division	1.05	1.14	19	3.05	$p<.01$**
All four operations	3.30	5.22	19	2.83	$p<.05$*

*Significant at .05

**Significant at .01

Remark: Reject the null hypothesis 1.2a in every specified case of operations, except addition.

Table 6.3

The Analysis of the Skill Test Results for School B

Operation Tested	$\bar{d}$	sd	df	t	p
Addition	.44	2.02	24	1.09	$.2<p<.3$
Subtraction	.24	1.42	24	.84	$.4<p<.5$
Multiplication	.40	1.19	24	1.68	$.1<p<.2$
Division	.96	1.67	24	2.87	$p<.01$**
All four operations	2.04	4.54	24	2.25	$p<.05$*

*Significant at .05

**Significant at .01

Remark: Reject the null hypothesis 1.2b on division and all four operations combined.

Table 6.4

The Analysis of the Skill Test Results for Both Schools

Operation Tested	$\bar{d}$	sd	df	t	p
Addition	.31	1.79	44	1.16	$.2<p<.3$
Subtraction	.49	1.47	44	2.23	$p<.05$*
Multiplication	.78	1.81	44	2.89	$p<.01$**
Division	1.02	1.62	44	4.24	$p<.01$**
All four operations	2.60	4.84	44	3.60	$p<.01$**

*Significant at .05

**Significant at .01

Remark: Reject the null hypothesis 1.2c in every case, except addition.

Conclusion

For School A, there are significant differences in the skills of the students in the use of the four fundamental operations combined, and each of the three operations, namely, subtraction, multiplication, and division, between the two testing results before and after the unit instruction, as measured by the standardized test of arithmetic computation.

For School B, there are significant differences in the skills of the students in the use of the four fundamental questions combined and the operation of division between the two testing results before and after the unit instruction, as measured by the standardized test of arithmetic computation.

For both schools combined, there are significant differences in the skills of the students in the use of all four fundamental operations combined, and each of the three operations, namely, subtraction, multiplication, and division, between the two testing results before and after the unit instruction, as measured by the standardized test of the arithmetic computation.

1.3 Problem Restated

Do the students who study the unit in basic inferential statistics develop problem solving methods, as measured by the Achievement Test Part II, reaching or exceeding the criterion referenced measurement of the test?

1.3a For the students in School A
1.3b For the students in School B
1.3c For the students in both schools.

Analysis Procedure

The criterion referenced procedure, as already described in the analysis procedure for Problem 1.1, was also employed for Problem 1.3.

Results

The answers to the three problems 1.3a, 1.3b, and 1.3c, analyzed from the results of Achievement Test Part II illustrated in Appendix I, and incorporated with the criterion referenced scores set for the test, are reported as follows:

a. For School A, 14 out of 21 students achieved the criterion referenced score of 7 or 50% of the total. The mean and standard deviation of the students' scores are 8.48 and 4.23.

b. For School B, 14 out of 24 students achieved the criterion referenced score of 7. The mean and standard deviation of the students' scores in this school are 7.04 and 3.30.

c. For both schools, 28 out of 45 students achieved the criterion referenced score of 7. The mean and standard deviation of the students' scores for both schools from the results of Achievement Test Part II are 7.71 and 3.79.

Conclusion

The criterion referenced numbers of students who achieved the criterion referenced score of 7 on the Achievement Test Part II are: 14 students for School A, 13 students for School B, and 27 students for both schools. Therefore the three results as illustrated in a, b, and c all have met the criterion reference for the feasibility assessment.

Consequently, with respect to the test employed and the criteria set, we conclude that the instruction of the unit in basic inferential statistics does contribute to the development of the problem solving method for the students.

1.4 Problem Restated

Do the students who study the unit in basic inferential statistics express some observable behavior which indicates they are developing habits of precise thought and expression?

Analysis Procedure

The results are descriptively analyzed with respect to the students' behavior as observed in the classroom and the results of the Conception Test.

Results

The answers to problem 1.4 are described with respect to both schools.

From the classroom observations: It was observed that the students had a vocabulary including statistical terms and symbols necessary to communicate among themselves and with the teacher during the course of the unit instruction. Usually the students used correct statistical terms in participating in discussion about the ideas with the teacher. They listened carefully and expressed no objections to the use of statistical terms and symbols by the teacher. And during the later stages of the unit instruction the use of various statistical terms and symbols involved in the process of instruction was common in the classroom. The students seemed to accept those statistical terms and symbols, incorporating them in their thoughts and expressions.

From the Conception Test results: Upon the completion of the unit instruction, the Conception Test was administered to measure the students' post anchoring ideas about statistical concepts. The results indicated that the students expressed more precise ideas or reasons than on the pretest for just about all of those statistical concepts selected, although some of those ideas expressed remained the same. The students used the statistical terms mean, median, and mode, as well as the symbols for those terms, to express their ideas about the measures of central tendency.

About the statistical concept involved in using a mean of a different size sample to estimate a population mean, the students expressed a lot of difficulty in giving the reasons for their selected responses in the pretest. In the post-test, however, those difficulties seemed to have disappeared as 45% of the students got the right answer with the very precise expressed reason that "a mean from a bigger size sample is a better estimate of a population mean."

About the statistical concept of using a sample mean as the estimate of a population mean, about 19% of the students in the pretest expressed their concerns of getting a mean from a sample by including only the extreme heights of the one hundred girls chosen. From the post-test result, however, none of them expressed this idea again. They seemed to demonstrate careful thought about the little chance for such an occurrence in a general procedure after they studied the unit.

Conclusion

From both classroom observations and the analysis of the students' pre- and post-test anchoring ideas, there is evidence of the students' development of habits of precise thought and expressions as a result of the unit instruction. This is indicative of the unit contribution to the fourth objective of junior high school mathematics.

1.5 Problem Restated

Do the students who study the unit in basic inferential statistics develop an understanding of the application of statistics, as measured by the subset of the Achievement Test Part I, reaching or exceeding the criterion referenced measurement of the test?

1.5a For the students in School A
1.5b For the students in School B
1.5c For the students in both schools.

Analysis Procedure

The criterion referenced procedure used for Problem 1.1 and 1.3 is also employed for Problem 1.5. The criterion references for the feasibility assessment employed for Problem 1.5 are: 14 out of 21 students achieved the criterion referenced score of 9 or 50% of the total in School A, 13 out of 24 students in School B and 27 out of 45 students in both schools achieved the same criterion referenced score of 9.

Results

Taking information from the results of the subset of the Achievement Test Part I, those test items categorized under the level of "application," illustrated in Appendix I, and incorporated with the criterion referenced measurement of the test, the answers to the three problems 1.5a, 1.5b, and 1.5c are analyzed and presented as follows:

a. For School A, 14 out of 21 students achieved the criterion referenced score of 9 or 50% of the total. $\bar{x} = 11.71$ and SD = 5.73 for the students' scores.

b. For School B, 16 out of 24 students achieved the criterion referenced score of 9. $\bar{x} = 11.25$ and SD = 3.99 for the students' scores in School B.

c. For both schools, 30 out of 45 students achieved the criterion referenced score of 9. In both schools, $\bar{x}$ and SD of the students' scores are 11.47 and 4.83.

Conclusion

School A, School B, and both schools combined all meet the criterion reference for the feasibility assessment, namely, the criterion referenced scores of the test and the criterion referenced number of students as set.

Hence, the students who study the unit in inferential statistics do develop an understanding of the application of statistics, as measured by the subset of the Achievement Test Part I, reaching or exceeding the criterion referenced score set for the test.

2. Feasibility Assessment of the Student Reactions

2.1 Descriptive Analysis of Questionnaires

Problem Restated

How do the students, after having studied the unit in basic inferential statistics, react to the unit as measured by the Student Reaction Test on the following variables?

2.11 Enjoyment of activities for each and both of the two schools
2.12 Difficulty of the subject for each and both of the two schools
2.13 Interest of the content for each and both of the two schools
2.14 Belief of new knowledge gained for each and both of the two schools
2.15 Perceived usefulness of statistics for each and both of the two schools
2.16 All five variables above for each and both of the two schools.

Analysis Procedure

For each variable in the test, the number and also the percent of the responses for each of the five scale answers are presented; the mean and standard deviation are also reported.

Results

Table 6.5 illustrates the test results for the problems 2.11, 2.12, 2.13, 2.14, 2.15, 2.16. The table gives the percentages and the numbers of the student responses to each specified variable on each of the five scale answers equivalent to 1, 2, 3, 4 and 5. The higher scale number indicates the higher degree of favorable response to the unit with respect to a specified variable. To respond in the scale of 5 for the variable "difficulty" means "strongly agree" that the unit is "not difficult." The table also presents the means and standard deviations of the scores for all variables.

Table 6.5

Percentages, Numbers, Means and Standard Deviations of the Responses from the Students' Reactions

		Number and Percent of Responses on Five Point Scale*						
Variable	School	1	2	3	4	5	$\bar{x}$	SD
Enjoyment	A	5 (4.76)	12 (11.43)	20 (19.05)	47 (44.76)	21 (20.00)	3.64	1.07
	B	37 (27.41)	40 (29.63)	27 (20.00)	21 (15.56)	10 (7.41)	2.46	1.25
	A & B	42 (17.50)	52 (21.67)	47 (19.58)	68 (28.33)	31 (12.92)	2.98	1.31
Difficulty	A	2 (1.90)	17 (16.19)	15 (14.29)	66 (62.86)	5 (4.76)	3.52	0.88
	B	9 (6.67)	35 (25.93)	24 (17.78)	61 (45.19)	6 (4.44)	3.15	1.06
	A & B	11 (4.58)	52 (21.67)	39 (16.25)	127 (52.92)	11 (4.58)	3.31	1.01
Interest	A	6 (5.71)	25 (23.81)	24 (22.86)	43 (40.95)	7 (6.67)	3.19	1.05
	B	38 (28.15)	46 (34.07)	20 (14.81)	22 (16.30)	9 (6.67)	2.39	1.24
	A & B	44 (18.33)	71 (29.58)	44 (18.33)	65 (27.08)	16 (6.67)	2.74	1.22
Knowledge gained	A	0 (0.00)	2 (1.90)	7 (6.67)	72 (68.57)	24 (22.86)	4.12	0.60
	B	2 (1.48)	6 (4.44)	15 (11.11)	87 (64.44)	25 (18.52)	3.94	0.78
	A & B	2 (0.83)	8 (3.33)	22 (9.17)	159 (66.25)	49 (20.42)	4.02	0.71
Usefulness perceived	A	4 (3.81)	6 (5.71)	23 (21.90)	54 (51.43)	18 (17.14)	3.72	0.94
	B	16 (11.85)	30 (22.22)	41 (30.37)	40 (29.63)	8 (5.93)	2.96	1.11
	A & B	20 (8.33)	36 (15.00)	64 (26.67)	94 (39.17)	26 (10.83)	3.29	1.23
All five variables	A	17 (3.24)	62 (11.81)	89 (16.95)	282 (53.71)	75 (14.29)	3.64	0.97
	B	102 (15.11)	157 (23.26)	127 (18.81)	231 (34.22)	58 (8.59)	2.98	1.23
	A & B	119 (9.92)	219 (18.25)	216 (18.00)	513 (42.75)	133 (11.08)	3.27	1.17

*The five point scale answers for the variables, except "difficulty," mean: 1: strongly disagree; 2: disagree; 3: undecided; 4: agree; 5: strongly agree.

For the variable "difficulty": 1: strongly agree; 2: agree; 3: undecided; 4: disagree; 5: strongly disagree.

Conclusion

2.11 Enjoyment of activities

For School A, 44.76% of the responses rated "agree" and 20% of the responses rated "strongly agree." The mean is 3.64 which is higher than the middle scale, 3.

For School B, 29.63% of the responses rated "disagree" and 27.41% of all responses rated "strongly agree." The mean is 2.46, well below the middle scale.

For both schools, 17.50% and 21.67% went to the categories of "disagree" and "strongly disagree"; while 28.33% and 12.92% went to the two higher categories. The mean is 2.98, about the middle scale.

With respect to the variable "enjoyment of activities," the students in School A reacted favorably to the unit; the students in School B reacted in the opposite direction—unfavorably to the unit. Therefore, when the responses for both schools are combined, the students' reaction can be rated neither favorable nor unfavorable.

2.12 Difficulty of the subject

For School A, about 67% of all the responses went to the categories "disagree" and "strongly disagree" that the unit is difficult. The mean is 3.52.

For School B, about 50% of all the responses rated "disagree" and "strongly disagree" about the difficulty of the subject. The mean is 3.15.

For both schools, about 57% of all responses rated "disagree" and "strongly disagree" about the difficulty of the unit.

The mean is 3.31.

For each and both of the schools, the students reacted favorably to the unit with respect to the variable "difficulty of the subject."

2.13 Interest of the content

For School A, about 29% of the responses went to the two "disagree" categories, and about 47% went to the two "agree" categories. The mean is 3.19, above the middle scale.

For School B, about 62% of the responses went to the two "disagree" categories; and the mean is 2.39, well below the middle scale.

For both schools, about 48% of the responses went to "disagree" and "strongly disagree"; and about 34% went to the two higher scales of answer. The mean is 2.74.

With respect to the variable "interest of the content," only the students in School A reacted favorably to the unit, while the students in School B and both schools combined did not.

2.14 Belief of new knowledge gained

For School A, about 91% of the responses went to "agree" and "strongly agree." The mean is 4.12 and standard deviation is 0.60.

For School B, about 83% of the responses went to "agree" and "strongly agree." The mean is 3.94 and standard deviation is 0.78.

For both schools, about 87% of the responses went to the two higher scale answers. The mean is 4.09 and standard deviation is 0.71.

For each and both of the two schools, the students reacted heavily in favor of the unit. The means are well above the middle scale 3; and all three standard deviations are less than 1, which illustrates the homogeneous opinions of the students with respect to this variable.

2.15 Perception of usefulness of statistics

For School A, about 68% of the responses went to "agree" and "strongly agree." The mean is 3.72.

For School B, about 34% of the responses went to "disagree" and "strongly disagree"; and about 35% of the responses went to the two higher scale answers. The mean is 2.96.

For both schools combined, about 23% went to "disagree" and "strongly disagree" and 50% responded "agree" and "strongly disagree." The mean is 3.29.

In School A, the students reacted favorably to the unit. In School B, the students reacted neither favorably nor unfavorably. However, for both schools combined the students reacted favorably to the unit.

2.16 All five variables

These combined five variables represent the students' opinions from the twenty five questionnaire items caregorized into five five-item groups under the five variables mentioned. The following conclusions illustrate the students' overall opinions about the unit.

For School A, about 68% of the responses illustrated opinions in favor of the unit; only about 15% illustrated opposite opinions. The mean is 3.64, well above the middle scale; the standard deviation is .97.

For School B, about 43% of the responses illustrated favorable opinions of the unit, while about 38% illustrated opposite opinions. The mean is about middle scale, 2.98.

For both schools combined, about 54% of the responses illustrated favorable opinions of the unit, and about 28% illustrated opposite opinions. The mean is over middle scale, 3.27.

Overall opinions of the students for School A and both schools combined are in favor of the unit, while for School B they are neither favorable nor unfavorable to the unit.

2.2 Significance of the Student Reactions Tested

Null Hypothesis Restated

There are no differences in the numbers of responses between "favorable" or "F" and "unfavorable" or "U" on the student reactions to the unit as measured by the Student Reaction Test with respect to the following variables:

2.2.1 Enjoyment of activities, for each and both of the schools
2.2.2 Difficulty of the subject, for each and both of the schools
2.2.3 Interest of the content, for each and both of the schools
2.2.4 Belief of new knowledge gained, for each and both of the schools
2.2.5 Perception of usefulness of statistics, for each and both of the schools
2.2.6 All five variables combined, for each and both of the schools.

Analysis Procedure

The chi-square goodness of fit procedure was employed to test the null hypotheses. The significance level of .05 is set for the hypothesis testing. Some terminology involved in the process of hypothesis testing in this study is defined as follows:

"F" or "Favorable" refers to the responses rating 4 and 5 in the answering scale, adding half the number of responses rating 3, on a specified variable.

"U" or "Unfavorable" refers to the responses rating 1 and 2 in the scale, and adding half the number of responses rating 3, on a specified variable.

"O" or "Observed scores" are the two values: F and U.

"E" or "Expected scores" are the two equal values each of which is half of F + U.

Results

Table 6.6 illustrates the results of testing the null hypotheses 2.2.1, 2.2.2, 2.2.3, 2.2.4, 2.2.5 and 2.2.6. The table gives the value of chi-square (χ^2) in testing the differences between the responses of "Favorable (F)" and "Unfavorable (U)," and also the probability of the observed chi-square.

Conclusions

2.2.1 Enjoyment of activities

For School A, the students indicated a significantly greater number of favorable responses than unfavorable ones.

For School B, the students indicated significantly less favorable responses than unfavorable ones.

For both schools combined, the students indicated

Table 6.6

Significance of the Responses from the Student Reaction Test

Variable	School	F U	Expected V. E	Observed V. O	df	χ^2	p
Enjoyment	School A	F	52.5	78	1	24.77	<.005**
		U	52.5	27			
	School B	F	67.5	44.5	1	15.67	<.005**(-)
		U	67.5	90.5			
	A & B	F	120	122.5	1	0.10	.7<p<.8
		U	120	117.5			
Difficulty	School A	F	52.5	78.5	1	25.75	<.005**
		U	52.5	26.5			
	School B	F	67.5	79	1	3.92	<.05*
		U	67.5	56			
	A & B	F	120	157.5	1	23.44	<.005**
		U	120	82.5			
Interest	School A	F	52.5	62	1	3.44	.05<p<.1
		U	52.5	43			
	School B	F	67.5	41	1	20.81	<.005**(-)
		U	67.5	94			
	A & B	F	120	103	1	4.82	<.05*(-)
		U	120	137			
New knowledge gained	School A	F	52.5	99.5	1	84.15	<.005**
		U	52.5	5.5			
	School B	F	67.5	119.5	1	80.12	<.005**
		U	67.5	15.5			
	A & B	F	120	219	1	163.35	<.005**
		U	120	21			
Usefulness perceived	School A	F	52.5	83.5	1	36.61	<.005**
		U	52.5	21.5			
	School B	F	67.5	68.5	1	0.03	.8<p<.9
		U	67.5	66.5			
	A & B	F	120	152	1	17.07	<.005**
		U	120	88			
All five variables	School A	F	262.5	401.5	1	147.21	<.005**
		U	262.5	123.5			
	School B	F	337.5	352.5	1	1.33	.2<p<.3
		U	337.5	322.5			
	A & B	F	600	754	1	79.05	<.005**
		U	600	446			

no significant difference in numbers between those of the favorable responses and the unfavorable ones.

2.2.2 Difficulty of the subject

For each and both of the two schools, the students indicated a significantly greater number of favorable responses than unfavorable ones.

2.2.3 Interest of content

For school A, the students' responses indicated no significant difference in numbers between those of the favorable and the unfavorable responses.

For school B, and the two schools combined, the students indicated significantly less favorable responses than unfavorable ones.

2.2.4 Belief of new knowledge gained

For each and both of the two schools, the students indicated a significantly greater number of favorable responses than unfavorable ones.

2.2.5 Perceived usefulness of statistics

For School A and the two schools combined, the students indicated a significantly greater number of favorable responses than unfavorable ones.

For School B, the students' responses indicated no significant difference in numbers between those of the favorable and the unfavorable responses.

2.2.6 All five variables combined

For School A and the two schools combined, the students indicated a significantly greater number of favorable responses than unfavorable ones.

For School B, the students' responses indicated no significant differences in numbers between those of the favorable and the unfavorable responses.

2.3 The Student Comments

The students were asked to make optional comments on various aspects of the unit as the last part of the Student Reaction Test. The list of all comments is presented in Appendix K. Following are the summaries of those comments.

For School A, of the fifteen students who made comments about the unit, only two of these made negative comments. Among the remaining thirteen students, four of those were very satisfied. As one student said, "I really like the course. It taught me something that I did not know and helped me to understand how statistics can be used and ran in everyday life." Six of those who also commented in favor of the unit would like to "have more time" or "go slower" in teaching the unit, in particular, near the end of the unit instruction. The two students who made the negative comments also put down the cause to the instructional time which "went too fast."

For School B, of the seventeen students who made comments about the unit, only two of those really liked and enjoyed it. One student said, "I liked it very much it was fun doing it." Among the remaining fifteen students, eight of these commented neutrally, pinpointing both like and dislike aspects, such as two students who said, "nice to learn but not consider this line of work." Two complained about homework; two said "enjoy but difficult," and one student commented about taking too much time for the early lessons and rushing too fast at the end. Seven students commented unfavorably about the unit; four of these said "Boring."

Conclusion

The students in School A expressed more favorable comments to the unit, however, a number who made those comments also complained about the time factor. The students in School B, on the other hand, made comments categorized as more negative reactions to various aspects of the unit.

3. The Teacher Reactions to the Unit

Problem Restated

What are the teachers' opinions about the unit in basic inferential statistics in relation to the following areas?

3.1 Perception of the unit contributions to the five objectives of junior high mathematics as listed:
  a. To develop an understanding of mathematical concepts and appreciation of mathematical structure
  b. To develop skill in the use of the fundamental process
  c. To develop systematic methods of analyzing problems and of presenting their solutions
  d. To develop habits of precise thought and expression
  e. To develop an understanding of the significance and application of mathematics in the modern world.

3.2 Perception of the instructional suitability of the unit.

3.3 Perception of the demands on the teacher's subject background.

3.4 Perception of the student reactions to the unit with respect to the five variables specified in the student reaction test.

3.5 Perception of the unit's being included in a regular mathematics program.

Results

The teachers' opinions about the unit in the above five areas are presented in Appendix L. Following is the summary of the two teachers' opinions.

3.1 The unit contributions to the objectives of junior high mathematics.

Teacher A: The objectives of mathematics are fulfilled by the unit of instruction, in vary degrees, depending on the student ability, interest, and background in mathematics.

Teacher B: The five objectives, as described in Problem 3.1 above, were served best by the unit of instruction in the order of a, c, d, e, b.

3.2 The instructional suitability of the unit.

Teacher A: Materials are satisfactory. There are both practical and theoretical applications involved. However, time allotment for the course was insufficient.

Teacher B: The instructional approach was good. She recommended having more experiments and activities performed by the students, especially for standard deviation.

As the unit was set up, the amount of teacher preparation was minimal. She recommended having a review every five lessons.

The instructional unit was not written in enough detail to be able to hand it out and let the students work on their own; the teacher had to be prepared enough to teach the basic ideas in each lesson. However, the students liked using their personal data in the lessons.

3.3 The demands on the teacher's subject background

Teacher A: She made the statement, "To be a success in any field of instruction, a teacher must have some background in the subject area." This seemed to indicate that she had enough background in statistics in order to teach the unit. Her perception of the general junior high mathematics teachers' background in statistics was not precisely stated—she said it varies with different teachers.

Teacher B: She had enough background in statistics to teach this type of instructional unit because she had one statistics course from university. All junior high mathematics teachers should have this background, because that one statistics course is required for a mathematics major in the local university.

3.4 The opinions on the student reactions to the unit.

Teacher A: "Interest was great. Students did work on their own time at home, some stayed after school hours and questioned me about various areas taken."

Teacher B: She assessed that the students enjoyed the activities, had some difficulty in understanding some of the concepts, and mostly did not perceive the usefulness of the subject. The course needed more practical examples involving life situations.

3.5 The opinions on the unit's inclusion in the regular math program.

Teacher A: The unit could be included in the mathematics program. It is dependent upon the student ability and the teacher's background. Only a limited portion of this course could be applied for those students who are weak in mathematics skills.

Teacher B: There is no extra time in the core program now to include this type of mathematics at grade nine level. However, if there was time, it would be an excellent supplementary or enrichment topic for grade nine or eight.

Conclusion

Both teachers expressed mostly favorable opinions in each of the five questioning areas. The unit of instruction did contribute in varying degrees to the fulfillment of the five objectives of mathematics in Alberta. The instructional suitability of the unit was assessed as satisfactory or good, and junior high mathematics teachers should have sufficient background to teach it. The student reactions of the unit were assessed as being positive. This type of unit could be included in the regular mathematics program upon the conditions of student ability and availability of sufficient time.

THE INVESTIGATION OF THE STUDENTS' ANCHORING IDEAS

1. Analysis of the Nature of the Students' Anchoring Ideas

Problem Restated

What are the students' anchoring ideas, as measured by the Conception Test before and after the instruction of the unit, about each of the following selected statistical concepts?

1.1 Measures of central tendency
1.2 The transformation of a mean, when a constant is added to the data
1.3 The transformation of a mean, when data are multiplied by a constant
1.4 A normal distribution
1.5 The use of a sample mean to estimate a population mean
1.6 Variability of scores
1.7 The transformation of a variance when a constant is added to the data
1.8 The effect of using a mean from a different size sample to estimate a population mean.

Results

Tables 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 6.13, and 6.14 give the answers to Problems 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, and 1.8. respectively. Each of these tables illustrates the students' answers or anchoring ideas with respect to a specified statistical concept, summarizing and arranging them in a certain ranking order. The numbers of students who expressed those anchoring ideas, with percentages in parentheses, for each and both of the two schools in both pre- and post-tests, are also presented in those tables. Table 6.10, giving the answer to Problem 1.4, also reports the numbers with percentages in parentheses of the examples that the students created to express the anchoring ideas about a normal distribution.

Table 6.7

The Students' Anchoring Ideas about Measures of Central Tendency

Question 1: The people in your mathematics class are each weighed and the weights are recorded. Your friend, who is not in the class, wants to know what the "middle" weight is. How would you answer this question?

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. Average or the idea of a mean	9 (52.94)	14 (56.00)	23 (54.76)	10 (58.82)	16 (64.00)	26 (61.90)
2. The idea of a median	0 (0.00)	3 (12.00)	3 (7.14)	5 (29.41)	5 (20.00)	10 (23.81)
3. The idea of a mode	1 (5.88)	1 (4.00)	2 (4.76)	1 (5.88)	2 (8.00)	3 (7.14)
4. The idea of a midpoint between the lowest and highest scores	3 (17.65)	4 (16.00)	7 (16.67)	1 (5.88)	0 (0.00)	1 (2.38)
5. Others	4 (23.53)	3 (12.00)	7 (16.67)	0 (0.00)	2 (8.00)	2 (4.76)

Table 6.8

The Students' Anchoring Ideas about the Transformation of a Mean when a Constant is Added to the Data

Question 2: The "mean" of a set of numbers is another name for the "average" of the set. For example, the mean of the set of numbers (2, 2, 3, 9, 10, 10) is six because the sum of the numbers divided by six is equal to six. If 123 is added to each of the numbers then the set consists of (125, 125, 126, 132, 133, 133). What is the mean of this set of numbers? How do you get the answer?

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. To get the correct answer by using transformation	4 (23.53)	4 (16.00)	8 (19.05)	9 (52.94)	11 (44.00)	20 (47.62)
2. To get the correct answer by observing the median of the new set	2 (11.76)	0 (0.00)	2 (4.76)	2 (11.76)	2 (8.00)	4 (9.52)
3. To get the correct answer by using the mean formula to work it out from the new set	4 (23.53)	7 (28.00)	11 (26.19)	5 (29.41)	7 (28.00)	12 (28.57)
4. To suggest working it out from the new set	3 (17.65)	9 (36.00)	12 (28.59)	1 (5.88)	4 (16.00)	5 (11.90)
5. The mean of the new set remains the same as the old one	3 (17.65)	2 (8.00)	5 (11.90)	0 (0.00)	1 (4.00)	1 (2.38)
6. Others	1 (5.88)	3 (12.00)	4 (9.52)	0 (0.00)	0 (0.00)	0 (0.00)

Table 6.9

The Students' Anchoring Ideas about the Transformation of a Mean when Data are Multiplied by a Constant

Question 3: The mean of the set of numbers (2, 2, 3, 9, 10, 10) is six. If each of the numbers in this set is multiplied by seven the set of numbers becomes (14, 14, 21, 63, 70, 70). What is the mean of this set? How do you get the answer?

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. To get the correct answer by using transformation	9 (52.94)	8 (32.00)	17 (40.48)	10 (58.82)	12 (48.00)	22 (52.38)
2. To get the correct answer by observing the median of the new set	1 (5.88)	0 (0.00)	1 (2.38)	1 (5.88)	1 (4.00)	2 (4.76)
3. To get the correct answer by using the mean formula to work it out from the new set	2 (11.76)	6 (24.00)	8 (19.05)	4 (23.53)	8 (32.00)	12 (28.57)
4. To suggest working it out from the new set	3 (17.65)	7 (28.00)	10 (23.81)	2 (11.76)	2 (8.00)	4 (9.52)
5. The mean remains the same	2 (11.76)	2 (8.00)	4 (9.52)	0 (0.00)	0 (0.00)	0 (0.00)
6. Others	0 (0.00)	2 (8.00)	2 (4.76)	0 (0.00)	2 (8.00)	2 (4.76)

Table 6.10

The Students' Anchoring Ideas about a Normal Distribution

Question 4: If the heights of all grade nine girls in Alberta were tabulated there would be a small number that were very tall and a small number that were very short. Most of the heights would be somewhere in the middle. Such an arrangement of numbers is known as a "normal distribution." What are some other examples of "normal distributions"?

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. To give examples of a normal distribution as related to some natural phenomena (numbers of students are given)	11 (64.71)	18 (72.00)	29 (69.05)	10 (58.82)	15 (60.00)	25 (59.52)
The number of examples involved:						
1.1 Weight	13 (43.33)	13 (34.21)	26 (38.24)	7 (18.92)	18 (33.33)	25 (27.47)
1.2 Intelligence	1 (3.33)	8 (21.05)	9 (13.24)	5 (13.51)	5 (9.26)	10 (10.99)
1.3 Size or length of a part or a whole of living creatures	9 (30.00)	6 (15.79)	15 (22.06)	8 (21.62)	10 (18.52)	18 (19.78)
1.4 Athletic ability of a person	0 (0.00)	0 (0.00)	0 (0.00)	3 (8.11)	1 (1.85)	4 (4.40)

Note: Students' answers items 1, 2, 3, 4, illustrate the numbers and percentages of the students for each and both of the two schools. All subitems, e.g. 1.1, 2.1, 3.1, illustrate the numbers and percentages of the examples given by the students.

Total examples: School A had 30 and 37 examples in pre- and post-tests.
School B had 38 and 54 examples in pre- and post-tests.

Table 6.10 (Continued)

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
2. To give examples of a normal distribution under certain specific conditions	5 (29.41)	3 (12.00)	8 (19.05)	5 (29.41)	6 (24.00)	11 (26.19)
The numbers of examples involved:						
2.1 Age, e.g. of all grade nine students in Alberta	2 (6.67)	3 (7.89)	5 (7.35)	2 (5.41)	9 (16.67)	11 (12.09)
2.2 Numbers of specified things or creatures in a certain area	0 (0.00)	0 (0.00)	0 (0.00)	6 (16.22)	2 (3.70)	8 (8.79)
2.3 Starting time for performing some specified activities, e.g. getting up time	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	6 (11.11)	6 (6.59)
2.4 Size or length or height of things	1 (3.33)	2 (5.26)	3 (4.41)	3 (8.11)	0 (0.00)	3 (3.30)
3. Others which cannot be justified as a normal distribution	1 (5.88)	4 (16.00)	5 (11.90)	2 (11.76)	1 (4.00)	3 (7.14)
3.1 Number of examples involved	4 (13.33)	6 (15.79)	10 (14.71)	3 (8.11)	3 (5.56)	6 (6.59)
4. Did not answer	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	3 (12.00)	3 (7.14)

Table 6.11

The Students' Anchoring Ideas About the Use of a Sample Mean to Estimate a Population Mean

Question 5: If 100 grade nine girls were picked from the schools in Alberta and their "mean height" was computed, would it be the same as the "mean height" of all grade nine girls in Alberta? Why or why not?

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. Negative responses "a sample mean is not equal to a population mean"						
1.1 Because of the variety of the girls' heights	2 (11.76)	10 (40.00)	12 (28.57)	2 (11.76)	6 (24.00)	8 (19.05)
1.2 Because this is just a small number of girls	1 (5.88)	3 (12.00)	4 (9.52)	3 (17.65)	9 (36.00)	12 (28.57)
1.3 Because of the chance of having the extreme heights of the 100 girls chosen	2 (11.76)	6 (24.00)	8 (19.05)	0 (0.00)	0 (0.00)	0 (0.00)
1.4 Because it is only an estimated guess or only the mean of a sample	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	2 (8.00)	2 (4.76)
1.5 Because the mean height of all grade nine girls is greater	1 (5.88)	1 (4.00)	2 (4.76)	0 (0.00)	1 (4.00)	1 (2.38)
1.6 Because you don't know how tall the hundred girls were	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)

Table 6.11 (Continued)

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
2. Positive responses or conditional responses						
2.1 It would be close because from a group of 100 girls there would be some tall, some short and some average	6 (35.29)	2 (8.00)	8 (19.05)	5 (29.41)	5 (20.00)	10 (23.81)
2.2 It would be quite close, because 100 is a large number you can get a good approximation	1 (5.88)	1 (4.00)	2 (4.76)	2 (11.76)	2 (8.00)	4 (9.52)
2.3 It would not be as accurate but gives the idea of a population mean. Need a bigger sample size	1 (5.88)	0 (0.00)	1 (2.38)	2 (11.76)	0 (0.00)	2 (4.76)
2.4 It would be close, but with less girls the mean height would be lower	0 (0.00)	0 (0.00)	0 (0.00)	1 (5.88)	0 (0.00)	1 (2.38)
2.5 It depends on the heights of the 100 girls chosen	3 (17.56)	1 (4.00)	4 (9.52)	1 (5.88)	0 (0.00)	1 (2.38)
3. No answer	0 (0.00)	0 (0.00)	0 (0.00)	1 (5.88)	0 (0.00)	1 (2.38)

Table 6.12

The Students' Anchoring Ideas About the Variability of Scores

Question 6: The mean for each of the following sets of numbers is six.

A = (2, 2, 3, 9, 10, 10)
B = (2, 5, 6, 6, 7, 10)
C = (2, 5, 5, 7, 7, 10)

A special number can be computed for set A as follows:

1. Subtract each number in the set from the mean of the set. For set A subtract each of the numbers from six. (See 1 below)
2. Square each of the differences. (See 2 below)
3. Sum these squares. (See 3 below)
4. Divide this sum by the number of squared terms that were added. (See 4 below)

1	2	3	4
$6 - 2 = 4$	$4^2 = 16$	16	$82 \div 6 = 13.7$
$6 - 2 = 4$	$4^2 = 16$	16	
$6 - 3 = 3$	$3^2 = 9$	9	
$6 - 9 = -3$	$(-3)^2 = 9$	9	
$6 - 10 = -4$	$(-4)^2 = 16$	16	
$6 - 10 = -4$	$(-4)^2 = 16$	16	
		82	

For set B, the special number is 5.7
Compute this special number for set C.
Can you see any relationship between these special numbers and the sets from which they were derived?

Table 6.12 (Continued)

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
1. The larger the difference of all numbers in the set, the greater the special numbers	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	1 (4.00)	1 (2.38)
2. The farther the numbers are away from the mean, the bigger the special numbers	1 (5.88)	0 (0.00)	1 (2.38)	1 (5.88)	2 (8.00)	3 (7.14)
3. When you get more higher or lower numbers in the set (when subtracted by six) you get higher special numbers	2 (11.76)	1 (4.00)	3 (7.14)	2 (11.76)	0 (0.00)	2 (4.76)
4. Numbers in the sets are not the same. Set B and C are close together, because there are 7 and 6 in the sets	2 (11.76)	1 (4.00)	3 (7.14)	3 (17.65)	1 (4.00)	4 (9.52)
5. More larger numbers make larger special numbers	4 (23.53)	2 (8.00)	6 (14.29)	1 (5.88)	3 (12.00)	4 (9.52)
6. More smaller numbers make larger special numbers	2 (11.76)	1 (4.00)	3 (7.14)	1 (5.88)	1 (4.00)	2 (4.76)
7. Only aware of the differences among the three given sets, but do not specify	6 (25.29)	19 (76.00)	25 (59.52)	4 (23.53)	10 (40.00)	14 (33.33)
8. Do not know or no answer	0 (0.00)	0 (0.00)	0 (0.00)	5 (29.41)	7 (28.00)	12 (28.57)

Table 6.13

The Students' Anchoring Ideas about the Transformation of a Variance when a Constant is Added to the Data

Question 7: The special number computed for the set (2, 2, 3, 9, 10, 10) was 13.7. If 123 were added to each of the numbers in the original set then what would the special number be for this new set?

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
1. The special number remains the same (13.7)	2 (11.76)	0 (0.00)	2 (4.76)	4 (23.53)	5 (20.00)	9 (21.43)
2. To suggest working it out from the new set	4 (23.53)	17 (68.00)	21 (50.00)	4 (23.53)	0 (0.00)	4 (9.52)
3. The special number is equal to the old one plus the constant, e.g. 136.7 by adding 123 to 13.7	10 (58.82)	6 (24.00)	16 (38.10)	8 (47.06)	10 (40.00)	18 (42.86)
4. To figure out a mean of the new set (129) as the answer	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	4 (16.00)	4 (9.52)
5. To get wrong answers by using unspecified procedures	1 (5.88)	0 (0.00)	1 (2.38)	1 (5.88)	1 (4.00)	2 (4.76)
6. Do not know or no answer	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	5 (20.00)	5 (11.90)

Table 6.14

The Students' Anchoring Ideas about the Effect of Using a Mean from a Different Size Sample to Estimate a Population Mean

Question 8: Imagine that you are given a box with six buttons in it. They are numbered 1, 1, 2, 6, 7 and 7. The "mean number" of this set is four. Now you are to imagine that you proceed according to instructions (a) and (b):

(a) Shake the box. Pick one button from the box. Record its number and then return the button to the box. Repeat this step so that you have two numbers recorded. Compute the mean of this set of two numbers.

(b) Pick a set of eight numbers by using the procedure in (a).

Compute the mean of this set of eight numbers.

Which of these two computed means, (a) or (b), is likely to be close to four (the mean of the six numbers in the box) or would it make no difference.

	Number (Percent) of Students					
	Pretest in School			Post-test in School		
Students' Answers	A	B	Both	A	B	Both
1. To select a mean of a sample with bigger size						
1.1 Because of a bigger sample	0 (0.00)	0 (0.00)	0 (0.00)	11 (64.71)	8 (32.00)	19 (45.24)
1.2 Because of the larger varieties, more chance to get a closer mean	7 (41.18)	7 (28.00)	14 (33.33)	2 (11.76)	3 (12.00)	5 (11.90)
1.3 Because it would be better	0 (0.00)	2 (8.00)	2 (4.76)	1 (5.88)	7 (28.00)	8 (19.05)
1.4 Because more numbers will get bigger mean	1 (5.88)	0 (0.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)
1.5 Because having more chance to get all number "six" than a small sample	1 (5.88)	0 (0.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)

Table 6.14 (Continued)

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
2. No difference						
2.1 It wouldn't make any difference	3 (17.56)	4 (16.00)	7 (16.67)	1 (5.88)	4 (16.00)	5 (11.90)
2.2 The mean would probably come out the same	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	1 (4.00)	1 (2.38)
2.3 It is just by chance	0 (0.00)	3 (12.00)	3 (7.14)	0 (0.00)	0 (0.00)	0 (0.00)
2.4 Not that much difference, the mean from a bigger size sample would be a little bit higher	1 (5.88)	0 (0.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)
3. To select a mean of a sample with smaller size						
3.1 Because it would be better	1 (5.88)	6 (24.00)	7 (16.76)	0 (0.00)	1 (4.00)	1 (2.38)
3.2 Because the largest sum of numbers would get smallest mean	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)
3.3 Because fewer numbers have better chance of getting the same number	0 (0.00)	1 (4.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)

Table 6.14 (Continued)

Students' Answers	Number (Percent) of Students					
	Pretest in School			Post-test in School		
	A	B	Both	A	B	Both
3.4 Because having less things to divide. "To pick 8 numbers" would have far more or far less average than the mean, 4.	1 (5.88)	0 (0.00)	1 (2.38)	1 (5.88)	0 (0.00)	1 (2.38)
3.5 Because more numbers, the mean is high	1 (5.88)	0 (0.00)	1 (2.38)	0 (0.00)	0 (0.00)	0 (0.00)
4. No answer or do not know	1 (5.88)	0 (0.00)	1 (2.38)	1 (5.88)	1 (4.00)	2 (4.76)

Conclusions

1. The students' anchoring ideas about a measure of central tendency (Table 6.7): About 55% of the students selected the idea of a mean in the pretest, and about 62% selected the same concept in the post-test. The idea of median was selected by about 7% in the pretest and 24% in the post-test. The idea of a midpoint between the highest and lowest scores was also illustrated by about 17% in the pretest but almost disappeared, 2%, in the post-test. The idea of a mode was not popular in either pre- or post-tests.

2. The students' anchoring ideas about the transformation of a mean when a constant is added to the data (Table 6.8): From the pretest results, about 29% suggested working it out from the new set, 26% managed to get the correct answer by working it out from the new set, and only 19% used the idea of transformation to get the correct answer. From the post-test results, about 48% used the idea of transformation to get the correct answer, about 29% used the formula working it out from the new set to get the correct answer, and about 12% still suggested working it out from the new set.

3. The students' anchoring ideas about the transformation of a mean when data are multiplied by a constant (Table 6.9): Many students used the idea of transformation to get the correct answer, about 40% in the pretest and 52% in the post-test. Others got the correct answers by using a formula (average) to work it out from the new set, about 19% and 2[illegible]% in pre- and post-test results. And some students suggested working it out from the new set, about 24% and 10% from pre- and post-test results.

4. The students' anchoring ideas about a normal distribution (Table 6.10): Many students described examples of a normal distribution relating to the natural phenomenon of distribution, such as weights of a large number of people, about 69% and 60% from the pre- and post-test results. Others gave examples of a normal distribution by justifying a specific set of data which may not be normal in general, 19% in pretest and 26% in the post-test. The rest of the students' answers went to the categories of not justified to be a normal distribution, and no expressed answers.

The students produced 68 examples to describe a normal distribution in the pretest, and 91 examples in the post-test. Among those examples in the pretest, 38% involved "weight," 22% involved "size or length of a part of a whole of a living creature," and the rest scattered. In the post-test, 27% involved "weight," 20% involved "size or length of a living creature," 12% involved "age," and the rest scattered.

5. The students' anchoring ideas about the use of a sample mean to estimate a population mean (Table 6.11): About 64% of the students in the pretest expressed negative responses such as, "a sample mean is not equal to a population mean," with a variety of reasons. Among those reasons, "because of the varieties of the girls' heights in a sample and a population" was expressed by about 29% of the students, and "because of the chance of having the extreme heights included in a 100 girl sample" was expressed by about 19% of the students. However, about 55% of the students in the post-test still expressed answers in the category of negative responses. And about

29% of the students gave the reason of too small a sample, "because this is just a small number of girls," and about 19% still expressed the reason "because of the varieties of the girls' heights."

About 36% of the students in the pretest went to the positive or conditional responses such as "would be close" and "it depends." Among these, 19% said "It would be close because from a group of 100 girls there would be some tall, some short and some average," and about 10% said "It depends on the heights of the 100 girls chosen." About 45% of the students in the post-test went to this category of responses. Among those, about 24% went to the answer that it would be close because there would be all types of girls' heights in a sample; about 10% went to the answer, "It would be quite close, because 100 is a large number you can get quite a good approximation"; and about 5% gave the answer "It would not be as accurate but gives the idea of a population mean."

6. The students' anchoring ideas about the variability of scores (Table 6.12): The students were expected to express their ideas of how the three given sets of data could be different, when these three sets of data have the same mean but different variances.

From the pretest results, about 60% of the students were aware of the differences among the three given sets of data, but failed to specify reasons; about 21% expressed the ideas that "more larger numbers in a set make a larger variance" and "more smaller numbers make a larger variance"; and only about 19% expressed the qualified answers, namely Items 1, 2, 3, and 4 illustrated in Table 6.12.

From the post-test results, about 33% of the students failed to specify the differences they perceived; about 29% did not answer and said that they did not know; about 24% expressed the qualified answers; and about 14% went to the answer of getting a larger variance because of either "more larger numbers" or "more smaller numbers."

7. The students' anchoring ideas about the transformation of a variance when a constant is added to the data (Table 6.13):

From the pretest results, 50% of the students suggested working it out from the new set; 38% expressed the idea that "the variance of a new set is equal to the variance of the old set plus the constant"; and only about 5% got the correct answer.

From the post-test results, about 43% of the students went to the idea of adding the constant to the old variance; about 21% got the correct answer; only about 10% still suggested working it out from the new set; and about 12% did not answer or said that they did not know.

8. The students' anchoring ideas about the effect of using a mean from a different size sample to estimate a population mean (Table 6.14):

From the pretest results, about 43% of the students chose a mean from a bigger size sample as the estimate of a population mean, and among those, about 33% gave the reason, "Because of the larger varieties, more chance to get a closer mean." About 29% of the students said, "It wouldn't make any difference whether a bigger or smaller size sample." And about 26% of the students selected a mean from a smaller size sample as the better estimate of a population mean.

From the post-test results, about 76% of the students selected a mean from a bigger size sample as the better estimate of a population mean, among those, 45% went to the reason "because of a bigger sample," 19% believed that it would be a better estimate, and 12% still expressed the reason as "the larger varieties, more chance to get a closer mean." About 14% still believed that it would not make any difference. Only about 5% still selected a mean from a smaller size sample. And about 5% did not express any ideas.

2. Analysis of Change between the Students' Pre and Post Anchoring Ideas

Problem Restated

How do the students in both School A and School B change their anchoring ideas between the pretest and post-test, as measured by the Conception Test, on the following statistical concepts?

2.1 Measures of central tendency
2.2 The transformation of a mean, when a constant is added to the data.
2.3 The transformation of a mean, when data are multiplied by a constant
2.4 A normal distribution
2.5 The use of a sample mean to estimate a population mean
2.6 Variability of scores
2.7 The transformation of a variance when a constant is added to the data
2.8 The effect of using a mean from a different size sample to estimate a population mean.

Analysis Procedure

The analysis procedure employed was developed as follows:

1. The first step was to categorize the students' anchoring ideas about each of those selected statistical concepts as illustrated in Tables 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 6.13, and 6.14.

The students' answers or anchoring ideas for each selected statistical concept were categorized into two main levels: S (satisfactory) and U (unsatisfactory) according to the quality of those available anchoring ideas. For some statistical concepts, S level was subcategorized into three sublevels: S^+ (high), S (medium), and S^- (low). Table 6.15 illustrates the definitions of the qualifications for the students' anchoring ideas established for the purpose of analysis in this study. The eight statistical concepts were ranked in the order of Concept 1, 2, and so on as those appeared in the statement of the problem. The item numbers of the students' anchoring ideas are the same as appeared in Tables 6.7-6.14.

Table 6.15

The Definitions of the Qualifications for the Students' Anchoring Ideas

Classified Level	Item Number of the Students' Anchoring Ideas for Each of Selected Concepts							
	C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8
S^+	1	1,2	1,2	1	2.1,2.2,2.3	1,2	1	1.1
S	2	3	3	2	1.1,1.2	3	2	1.2,1.3
S^-	3	4	4	-	1.3,2.4,2.5	4,5,6	3	1.4,1.5
U	4,5	5,6	5,6	3,4	The rest	7,8	4,5,6	The rest

2. The second step was to identify the format of the change of the students' anchoring ideas between the pre- and post-test results by using the classified levels: S^+, S, S^-, U. For example the format of US means that from the pretest the anchoring idea was in the level of U, but from the post-test the anchoring idea changed

to the level of S.

For the purpose of analysis procedure, the format of the change between the pre- and post-test results of the students' anchoring ideas are presented as the elements of the matrix of change illustrated in Table 6.16.

Table 6.16

The Matrix of the Changing Format between the Students' Pre and Post Anchoring Ideas

	S^+	S	S^-	U
S^+	S^+S^+	S^+S	S^+S^-	S^+U
S	SS^+	SS	SS^-	SU
S^-	S^-S^+	S^-S	S^-S^-	S^-U
U	US^+	US	US^-	UU

3. The third step was to record the results in the matrix, and to interpret the results of that transformed matrix.

With respect to the matrix in Table 6.16, each of its sixteen elements is replaced by a number, with percentage in parentheses, of the students who meet that particular changing format between their pre and post anchoring ideas about each of those selected statistical concepts. For instance, in the location of the element US, the number as well as percentage of the students who had anchoring ideas in the level of U from the pretest and in the level of S from the post-test is recorded.

Once the matrix is filled with the numbers and percentages of students as described above, the results can be interpreted, with reference to the matrix in Table 6.16, as follows:

3.1 Each element of the matrix illustrates the number and percentage of the students who meet that particular format of change.

3.2 The diagonal elements, S^+S^+, SS, S^-S^-, UU, each illustrates the number of students having no change in their anchoring ideas with respect to each level. The total number of students having no change in all levels can also be computed.

3.3 Excluding the diagonal elements, the matrix is divided into two halves, the upper half and the lower half. The upper half of the matrix illustrates the numbers of students who changed from a higher level to a lower one in various formats. The lower half of the matrix, on the other hand, illustrates the numbers of students who changed from a lower level to a higher one. The total for each of those two types of change can also be computed.

3.4 Ignoring the pretest results, one can find the number of students in each level from the post-test by computing the sum of each column. Similarly, the sum of each row illustrates the number of students in each level from the pretest.

4. The fourth step was to create a format prepared for the presentation of the results. Table 6.17, referring to the description in the third step, illustrates the format employed in presenting the results in the next section.

Table 6.17

The Format in Presenting the Results of the Changes between the Students' Pre and Post Anchoring Ideas

ΣD	S^+	S	S^-	U	ΣUH
S^+	S^+S^+	S^+S	S^+S^-	S^+U	ΣR_1
S	SS^+	SS	SS^-	SU	ΣR_2
S^-	S^-S^+	S^-S	S^-S^-	S^-U	ΣR_3
U	US^+	US	US^-	UU	ΣR_4
ΣLH	ΣC_1	ΣC_2	ΣC_3	ΣC_4	ΣT

4.1 ΣCi is the sum of each column of the matrix, equivalent to the number of students in the post-test for each level.

4.2 ΣRi is the sum of each row of the matrix, the number of students in the pretest for each level.

4.3 ΣDii is the sum of the diagonal elements of the matrix, the number of students who do not change between pre- and post-tests.

4.4 Excluding the diagonal in 4.3.

4.4.1 ΣUH is the sum of the upper half elements of the matrix, the number of students who change from high to low levels.

4.4.2 ΣLH is the sum of the lower half elements of the matrix, the number of students who change from low to high levels.

4.5 ΣT is the total number of students who took the test, which is equal to $\Sigma\Sigma Ci$ or $\Sigma\Sigma Ri$.

Results

Tables 6.18, 6.19, 6.20, 6.21, 6.22, 6.23, 6.24 and 6.25 illustrate the results of Problems 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, and 2.8 respectively. These tables were developed by using data from the initial analysis procedure, in particular, Tables 6.15 and 6.17, which in turn are related to the list of the students' anchoring ideas for each of the eight selected statistical concepts as illustrated in Tables 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 6.13, and 6.14. The results of the Conception Test for individual students, as illustrated in Appendix J, were also used to complete those tables—6.18 to 6.25.

Each of these tables, 6.18 to 6.25, gives the numbers along with the percentages of the students from both schools with respect to the various expected changing formats between the students pre- and post-test anchoring ideas. The numbers of students who did not change their anchoring ideas between pre- and post-tests, and who changed from a low level to a high one and from high to low levels are reported. In addition, the numbers of students who achieved each of the four specified levels, S^+, S, S^-, U in both pre- and post-tests are presented.

Table 6.18

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on Measures of Central Tendency

24 (57.14)	S^+	S	S^-	U	5 (11.90)
S^+	18 (42.86)	4 (9.52)	1 (2.38)	0 (0.00)	23 (54.76)
S	1 (2.38)	2 (4.76)	0 (0.00)	0 (0.00)	3 (7.14)
S^-	1 (2.38)	0 (0.00)	1 (2.38)	0 (0.00)	2 (4.76)
U	6 (14.29)	4 (9.52)	1 (2.38)	3 (7.14)	14 (33.33)
13 (30.95)	26 (61.90)	10 (23.81)	3 (7.14)	3 (7.14)	42 (100.00)

Conclusions

Table 6.18 indicates the change between the students pre and post anchoring ideas on measures of central tendency as follows (refer also to Tables 6.7, 6.15, 6.17):

1. 57.14% of the students did not change their ideas on this concept; among these 42.86% used the idea of a mean or average or S^+ level in both pre- and post-tests.

2. 30.95% of the students changed their ideas from low to high levels; the majority of those, 14.29% changed from U level to S^+ level, and 9.52% from U level to S level or the idea of a median.

3. 11.90% of the students changed their ideas from high to low levels, the majority of those 9.52% changing from S^+ to S level or from the idea of a mean or average to the idea of a median.

4. With respect to the four specified levels, S^+, S, S^-, and U, the numbers of the students in both pre- and post-tests were:

    4.1 For pretest 54.76%, 7.14%, 4.76%, 33.33%.

    4.2 For post-test 61.90%, 23.81%, 7.14%, 7.14%.

Table 6.19

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on the Transformation of a Mean, when a Constant is Added to the Data

13 (30.95)	S^+	S	S^-	U	4 (9.52)
S^+	8 (19.05)	2 (4.76)	0 (0.00)	0 (0.00)	10 (23.81)
S	7 (16.67)	3 (7.14)	1 (2.38)	0 (0.00)	11 (26.19)
S^-	4 (9.52)	4 (9.52)	2 (4.76)	1 (2.38)	11 (26.19)
U	5 (11.90)	5 (11.90)	0 (0.00)	0 (0.00)	10 (23.81)
25 (59.52)	24 (57.14)	14 (33.33)	3 (7.14)	1 (2.38)	42 (100.00)

Conclusions

Table 6.19 indicates the change between the students' pre and post anchoring ideas on the transformation of a mean when a constant is added to the data as follows (refer also to Tables 6.8, 6.15, 6.17):

1. 30.95% of the students did not change their ideas on this concept between pre- and post-tests, the majority, 19.05%, already had the correct idea from the pretest and remained in the S^+ level for the post-test.

2. 59.52% of the students changed their ideas from lower to higher levels. Among those, 16.67% changed from S to S^+ levels, 11.90% each from U to S^+ levels and from U to S levels, and 9.52% each from S^- to S^+ levels and from S^- to S levels.

3. 9.52% of the students changed from high to low levels. The majority of those, 4.76%, changed from S^+ to S level or from "to get the answer by transformation" to "to get the answer by working it out from the new set."

4. The numbers of students in pre- and post-tests, with respect to the four specified levels S^+, S, S^-, and U were:

   4.1 23.81%, 26.19%, 26.19%, 23.81% for pretest.

   4.2 57.14%, 33.33%, 7.14%, 2.38% for post-test.

Table 6.20

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on the Transformation of a Mean, when Data are Multiplied by a Constant

19 (45.24)	S^+	S	S^-	U	6 (14.29)
S^+	13 (30.95)	4 (9.52)	1 (2.38)	0 (0.00)	18 (42.86)
S	4 (9.52)	3 (7.14)	0 (0.00)	0 (0.00)	7 (16.67)
S^-	4 (9.52)	4 (9.52)	2 (4.76)	1 (2.38)	11 (26.19)
U	3 (7.14)	1 (2.38)	1 (2.38)	1 (2.38)	6 (14.29)
17 (40.48)	24 (57.14)	12 (28.57)	4 (9.52)	2 (4.76)	42 (100.00)

Conclusions

Table 6.20 illustrates the change between the students' pre and post anchoring ideas on the transformation of a mean, when data are multiplied by a constant as follows (refer to Tables 6.9, 6.15, 6.17):

1. 42.24% of the students did not change their ideas on this concept between pre- and post-tests. Among those, 30.95% were in the highest level, S^+.

2. 40.48% of the students changed from low to high levels. Among those, 9.52% each changed from S to S^+, from S^- to S^+, and S^- to S; and 7.14% changed from U to S^+.

3. 14.29% of the students changed their ideas from high to low level. The majority of those, 9.52%, went to the change from S^+ to S.

4. The number of students in each of the four levels S^+, S, S^-, and U were:

   4.1 42.86%, 16.67%, 26.19%, 14.29% for the pretest.

   4.2 57.14%, 28.57%, 9.52%, 4.76% for the post-test.

Table 6.21

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on a Normal Distribution

27 (64.29)	S^+	S	S^-	U	10 (23.81)
S^+	20 (47.62)	8 (19.05)	--	1 (2.38)	29 (69.05)
S	4 (9.52)	3 (7.14)	--	1 (2.38)	3 (19.05)
S^-	--	--	--	--	--
U	1 (2.38)	0 (0.00)	--	4 (9.52)	5 (11.90)
5 (11.90)	25 (59.52)	11 (26.19)	--	6 (14.29)	42 (100.00)

Conclusions

Table 6.21 illustrates the change between the students' pre and post anchoring ideas on a normal distribution as follows (refer also to Tables 6.10, 6.15, 6.17):

1. 64.29% of the students expressed no change in their ideas on this concept between pre- and post-tests. Among those, 47.62% remained in the S^+ level giving examples of a normal distribution relating to some natural phenomena.

2. 11.90% of the students expressed change from low to high levels, in which 9.52% were from S to S^+ levels citing examples of a normal distribution under certain specific conditions.

3. 23.81% of the students expressed change from high to low levels, in which 19.05% changed from S^+ to S level.

4. The numbers of students in each of the four levels S^+, S, S^-, and U were:

   4.1 69.05%, 19.05%, --, 11.90% for the pretest.

   4.2 59.52%, 26.19%, --, 14.29% for the post-test.

Note: S^- was not defined for the students' ideas on this concept.

Table 6.22

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on the Use of a Sample Mean to Estimate a Population Mean

17 (40.48)	S^+	S	S^-	U	8 (19.05)
S^+	6 (14.29)	3 (7.14)	1 (2.38)	1 (2.38)	11 (26.19)
S	4 (9.52)	10 (23.81)	0 (0.00)	2 (4.76)	16 (38.10)
S^-	5 (11.90)	5 (11.90)	1 (2.38)	1 (2.38)	12 (28.57)
U	1 (2.38)	2 (4.76)	0 (0.00)	0 (0.00)	3 (7.14)
17 (40.48)	16 (38.10)	20 (47.62)	2 (4.76)	4 (9.52)	42 (100.00)

Conclusions

Table 6.22 illustrates the change between the students' pre and post anchoring ideas on the use of a sample mean to estimate a population mean as follows (refer also to Tables 6.11, 6.15, 6.17):

1. 40.48% of the students expressed no change in their ideas between pre- and post-tests on the above concept. Among those, 23.81% and 14.29% remained in the levels of S and S^+ respectively.

2. 40.48% of the students expressed change from low to high levels. Among these, 11.90% each changed from S^- to S^+ and from S^- to S; and 9.52% changed from S to S^+.

3. 19.05% of the students expressed change from high to low levels. Among these, 7.14% changed from S^+ to S and 4.76% from S to U.

4. The numbers of students in each of the four levels S^+, S, S^-, and U were:

   4.1 26.19%, 38.10%, 28.57%, 7.14% for the pretest.

   4.2 38.10%, 47.62%, 4.76%, 9.52% for the post-test.

Table 6.23

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on the Variability of Scores

32 (76.19)	S^+	S	S^-	U	4 (9.52)
S^+	2 (4.76)	0 (0.00)	0 (0.00)	0 (0.00)	2 (4.76)
S	1 (2.38)	1 (2.38)	0 (0.00)	1 (2.38)	3 (7.14)
S^-	1 (2.38)	1 (2.38)	7 (16.67)	3 (7.14)	12 (28.57)
U	0 (0.00)	0 (0.00)	3 (7.14)	22 (52.38)	25 (59.52)
6 (14.29)	4 (9.52)	2 (4.76)	10 (23.81)	26 (61.90)	42 (100.00)

Conclusions

Table 6.23 illustrates the change between the students' pre and post anchoring ideas on the variability of scores as follows (refer also to Tables 6.12, 6.15, 6.17):

1. 76.19% of the students expressed no change in their ideas between the pre and post tests on the above concept. Among those, 52.38% remained in U level, 16.67% in S^- level, and 4.76% in S^+ level.

2. 14.29% of the students expressed change from low to high levels. Among those 7.14% changed from U to S^-; and 2.38% each from S^- to S^+, S^- to S, and S to S^+.

3. 9.52% of the students changed from high to low levels. The majority of those, 7.14%, changed from S^- to U.

4. The numbers of students in each of the four levels S^+, S, S^-, and U were:

   4.1 4.76%, 7.14%, 28.57%, 59.52% for the pretest.

   4.2 9.52%, 4.76%, 23.81%, 61.90% for the post-test.

Table 6.24

The Analysis of Change between the Students' Pre and Post Anchoring Ideas on the Transformation of a Variance when a Constant is Added to the Data

13 (30.95)	S^+	S	S^-	U	18 (42.86)
S^+	2 (4.76)	0 (0.00)	0 (0.00)	0 (0.00)	2 (4.76)
S	4 (9.52)	1 (2.38)	9 (21.43)	7 (16.67)	21 (50.00)
S^-	3 (7.14)	2 (4.76)	9 (21.43)	2 (4.76)	16 (38.10)
U	0 (0.00)	1 (2.38)	1 (2.38)	1 (2.38)	3 (7.14)
11 (26.19)	9 (21.43)	4 (9.52)	19 (45.24)	10 (23.81)	42(100.00)

Conclusions

Table 6.24 illustrates the change between the students' pre and post anchoring ideas on the transformation of a variance when a constant is added to the data as follows (refer also to Tables 6.13, 6.15, 6.17):

1. 30.95% of the students expressed no change in their ideas between pre- and post-tests on the above concept. Among those, 21.43% remained in S^-, and 4.76% in S^+.

2. 26.19% of the students expressed change of their ideas from low to high levels. Among those, 9.52% changed from S to S^+ and 7.14% from S^- to S^+.

3. 42.86% of the students changed their ideas from high to low levels. Among those, 21.43% changed from S to S^- and 16.67% from S to U.

4. The numbers of students in each of the four levels S^+, S, S^-, and U were:

   4.1 4.76%, 50.00%, 38.10%, 7.14% for the pretest.

   4.2 21.43%, 9.52%, 45.24%, 23.81% for the post-test.

Table 6.25

The Analysis of Change between the Students' Anchoring Ideas on the Effect of Using a Mean from a Different Size Sample to Estimate a Population Mean

13 (30.95)	S^+	S	S^-	U	2 (4.76)
S^+	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)
S	9 (21.43)	5 (11.90)	0 (0.00)	2 (4.76)	16 (38.10)
S^-	2 (4.76)	0 (0.00)	0 (0.00)	0 (0.00)	2 (4.76)
U	8 (19.05)	8 (19.05)	0 (0.00)	8 (19.05)	24 (57.14)
27 (64.29)	19 (45.24)	13 (30.95)	0 (0.00)	10 (23.81)	42 (100.00)

Conclusions

Table 6.25 illustrates the change between the students' pre and post anchoring ideas on the effect of using a mean from a different size sample to estimate a population mean as follows (refer also to Tables 6.14, 6.15, 6.17):

1. 30.95% of the students expressed no change in their ideas between pre- and post-tests on the above concept. From those students, 19.05% remained in U level and 11.90% in S.

2. 64.29% of the students expressed change in their ideas from low to high levels. Among those, 21.43% changed from S to S^+, and 19.05% each changed from U to S^+ and U to S.

3. 4.76% of the students changed from high to low levels, and those went to the change of S to U.

4. The numbers of students in each of the four levels S^+, S, S^-, and U were:

    4.1 0.00%, 38.10%, 4.76%, 57.14% in the pretest.

    4.2 45.24%, 30.95%, 0.00%, 23.81% in the post-test.

SUMMARY

The results of the study, categorized into three main parts—the constructibility of the unit, the feasibility assessment in using the unit, and the investigation of the students' anchoring ideas about statistical concepts before and after the unit instruction—have been presented. The trends and conclusions of the results are as follows:

1. The unit constructibility was justified on the basis of the analysis of the unit appropriateness for grade nine students and the five objectives of junior high school mathematics of Alberta.

2. The feasibility assessment in using the unit indicated positive results, in particular:

2.1 The use of the unit in teaching grade nine students is regarded as feasible with respect to the unit contribution to each of the five objectives of junior high school mathematics.

2.2 The use of the unit is regarded as feasible with respect to the students' overall reactions to the unit on five variables. However, the students did indicate negative reactions on the variable "Interest."

2.3 The use of the unit is regarded as feasible with respect to the opinions of the teachers who taught the unit.

3. The results of the investigation of the students' anchoring ideas on some selected statistical concepts before and after the unit instruction were analyzed in two ways:

3.1 A list of the students' anchoring ideas, along with the numbers and percentages of the students who expressed those ideas with respect to each of the eight selected statistical concepts were reported for each and both of the two schools in both pre- and post-tests.

3.2 The change between the students' pre and post anchoring ideas about each of those selected statistical concepts was analyzed by establishing a format of the expected change for those ideas. The results were reported with respect to the created format.

CHAPTER 7

SUMMARY, CONCLUSIONS AND DISCUSSION, AND RECOMMENDATIONS

INTRODUCTION

Chapter 7, the last chapter of the report, presents: a summary of the study, conclusions and discussion of the results, and recommendations resulting from the study. The summary consists of two parts: a summarized description of the purposes and design of the study, and a summary of the results of the study.

The conclusions and discussion of the results, presented as the second section of the present chapter, are divided into the following parts: the introduction as related to the three purposes of the study, the unit constructibility, the unit fulfillment of the five objectives of mathematics, the student reactions, the teacher reactions, the students' anchoring ideas, and limitations of the study.

As the last section of the chapter, recommendations resulting from this study are made in the following order: the unit inclusion in the mathematics program, the unit development, use and improvement of materials, the investigation of anchoring ideas, and recommendations for further study.

SUMMARY OF THE STUDY

Purpose, Design and Conduct of the Study

The purpose of this study was threefold: (1) To construct a unit in basic inferential statistics, (2) To determine the feasibility of using the unit to teach grade nine students, (3) To investigate the existence and change of the students' anchoring ideas about some selected statistical concepts before and after the unit instruction.

The problems to be answered in this study were established along the lines of its threefold purpose:

1. Can a unit in inferential statistics be constructed in such a way that it is appropriate for grade nine students and the five general objectives of Alberta junior high mathematics?

2. Is the use of the unit in inferential statistics in teaching grade nine students feasible with respect to each of the five objectives of junior high mathematics, the student reactions, and the teacher reactions?

3. What are the students' anchoring ideas about some selected statistical concepts before and after the unit instruction? And how do they change their ideas between pre and post unit testing?

After reviewing various influential reports advocating the inclusion of statistics in elementary and secondary schools, several related research projects done in this field, and theoretical background related to instruction, the unit in basic inferential statistics was designed and constructed to serve the first purpose of the study and to provide the material employed in teaching grade nine students.

Instruments were developed to gather data on various aspects of the specified dimensions of the unit feasibility assessment for the second purpose of the study. The Conception Test was also developed to collect data for the investigation of the existence and change of the students' anchoring ideas for the third purpose of the study.

The study was conducted by using two grade nine classes from two schools in the city of Edmonton, Alberta. The two classes were taught by their regular mathematics teachers and used the materials developed for the unit in basic inferential statistics. One school took eighteen forty-minute periods and the other took twenty-two forty-minute periods for teaching time during the period of about four months.

Upon the completion of the conduct of study, data on various aspects were collected and analyzed to answer the problems and to test the hypotheses relating to the threefold purpose of the study. The results of the study, along with the criteria of judgement, were reported. For the feasibility assessment, the criterion reference of 60% of the students in both schools was used as the number of students expected to achieve the criterion referenced score for the test.

Summary of Results

1. A unit in inferential statistics was constructible with respect to its appropriateness for grade nine students and the five objectives of junior high mathematics in the Province of Alberta.

2. The use of the unit in inferential statistics in teaching grade nine students was feasible with respect to the five objectives of junior high mathematics.

2.1 The students developed an understanding of statistical concepts.

2.1a For School A, 14 students, or 66.67% of the students, reached the criterion referenced score for the Achievement Test.

2.1b For School B, 13 students, or 54.17% of the students, reached the criterion referenced score for the Achievement Test.

2.1c For both schools, 27 students, or 60.00% of all students, reached the criterion referenced score for the Achievement Test.

2.2 The students developed skill in the use of the fundamental process.

2.2a For School A, there were significantly increased scores on the use of subtraction, multiplication, division, and the four operations combined as measured by the Standardized Test given before and after the course. However, there were no significantly increased scores with respect to addition.

2.2b For School B, there were significantly increased scores on the use of division, and the four operations combined, but no significantly increased scores with respect to the remaining operations.

2.2c For both schools, there were significantly increased scores on the use of subtraction, multiplication, division, and the four operations combined, but no significantly increased scores with respect to addition.

2.3 The students developed problem solving methods.

2.3a For School A, 14 students, or 66.67% of the students, reached the criterion referenced score for the Achievement Test Part II. dealing with the problem solving methods.

2.3b For School B, 14 students, or 58.33% of the students, reached the criterion referenced score for the test.

2.3c For both schools combined, 28 students, or 62.22% of all students, reached the criterion referenced score for the test.

2.4 The students developed habits of precise thought and expression. Observations made in the classroom during the unit instruction, and the ideas expressed by the students on the pre and post Conception Tests, indicated that the students in both schools exhibited behavior which showed development in the habits of precise thought and expression.

2.5 The students developed an understanding of the application of statistical concepts.

2.5a For School A, 14 students, or 66.67% of the students, reached the criterion referenced score for the Achievement Test dealing with application.

2.5b For School B, 16 students, or 66.67% of the students, reached the criterion referenced score for the test.

2.5c For both schools combined, 30 students, or 66.67% of all students, reached the criterion referenced score for the test.

3. The use of the unit in basic inferential statistics in teaching grade nine students was generally feasible with respect to the students' reactions to the unit instruction on each and all of the following variables: (1) Enjoyment of activities, (2) Difficulty of the subject, (3) Interest of the content, (4) Feeling of new knowledge gained, and (5) Perception of the usefulness of statistics.

3.1 For School A

The students reacted significantly in favor of the unit on Variables 1, 2, 4, 5, and all five combined.

The students did not react significantly in favor of the unit on Variable 3.

3.2 For School B

The students reacted significantly in favor of the unit on Variables 2 and 4.

The students did not react significantly in favor of the unit on Variables 5 and all five variables combined.

The students reacted significantly not in favor of the unit on Variables 1 and 3.

3.3 For both schools combined

The students reacted significantly in favor of the unit on Variables 2, 4, 5 and all five variables combined.

The students did not react significantly in favor of the unit on Variable 1.

The students reacted significantly not in favor of the unit on Variable 3.

4. The use of the unit in inferential statistics in teaching grade nine students was feasible with respect to the teachers' opinions. The two teachers expressed opinions agreeable to the unit in each of the following five areas: The unit served the five general objectives of Alberta junior high mathematics in varying degrees. The instructional suitability of the unit was satisfactory. Junior high school mathematics teachers, in general, should have sufficient subject background to teach the unit. The teachers perceived that the students reacted favorably to the unit. And the unit in statistics could be included in the regular school program.

5. The investigation of the students' anchoring ideas, before and after the unit instruction, about the following statistical concepts: (1) Measures of central tendency, (2) Variability of scores, (3) The effect on a mean when a constant is added to the data, (4) The effect on a mean when the data are multiplied by a constant, (5) The effect on a variance when a constant is added to the data, (6) A normal distribution, (7) The use of a sample mean as

the estimate of a population mean, and (8) The effect of using a mean from a different size sample to estimate a population mean, revealed the following summarized results:

For each of the above eight statistical concepts, the anchoring ideas most commonly expressed by the students in the pretest and post-test were:

5.1 Concept 1

Pre: 54.76% of the students expressed the idea of average or mean.

Post: 61.90% of the students expressed the same idea as above.

5.2 Concept 2

Pre: 59.52% of the students expressed the idea that they perceived the differences among the three given sets of scores but failed to relate the differences to the scores variability.

Post: 33.33% of them expressed the same idea as above.

5.3 Concept 3

Pre: 28.57% of the students expressed the idea of suggesting the computation of a new mean directly from the transformed data.

Post: 47.62% of the students expressed the idea of getting a correct new mean by transformation.

5.4 Concept 4

Pre: 40.48% of the students expressed the idea of getting a correct new mean by transformation.

Post: 52.38% of them expressed the same idea as above.

5.5 Concept 5

Pre: 50.00% of the students expressed the idea of suggesting the computation of a new variance directly from the transformed data.

Post: 42.86% of them expressed the idea of getting a new variance by adding the constant to the old variance.

5.6 Concept 6

Pre: 69.05% of the students expressed the idea of a normal distribution by illustrating it with examples which involved a natural phenomenon of distribution, such as weights of a large number of people.

Post: 59.52% of them expressed the same idea as above.

5.7 Concept 7

Pre: 28.57% of the students expressed the idea that "the mean height of one hundred girls is not equal to that of a population because of the varieties of the girls' heights."

Post: 28.57% of the students expressed the idea that "the mean height of one hundred girls is not equal to that of a population because of too small a number of girls in the sample."

5.8 Concept 8

Pre: 33.33% of the students expressed the idea that "the mean from a bigger sample is a better estimate of a population mean because of larger variety, more chance."

Post: 45.24% of them expressed the idea of preference for a mean from a bigger sample because the bigger the size of the sample mean, the better the estimate.

6. The analysis of change between the students' pre and post anchoring ideas on the eight statistical concepts, as described in 5, yielded the results as summarized in Table 7.1.

Table 7.1

Summarized Results for Analysis of Change between the Students' Pre and Post Anchoring Ideas

	Percentages of Students in Categories		
Concept No.	LH	NC	HL
1	30.95	57.14	11.90
2	14.29	76.19	9.52
3	59.52	30.95	9.52
4	40.48	45.24	14.29
5	26.19	30.95	42.86
6	11.90	64.29	23.81
7	40.48	40.48	19.05
8	64.29	30.95	4.76

Table 7.1 reports the number of students in percent who expressed ideas about each of those selected concepts which showed:

(1) No change of ideas (NC), (2) Change from lower quality ideas in the pretest to higher quality ideas in the post-test (LH), or (3) Change from higher quality ideas to lower ones (HL) between pretest and post-test.

CONCLUSIONS AND DISCUSSION

Introduction

Johnson (1973), in Definitions and Models in Curriculum Theory, defines a curriculum:

> A curriculum is a structured series of intended learning outcomes (p. 48).

From the above point of view, the set of five objectives for junior high mathematics in the Province of Alberta is a curriculum since the statements of the five objectives represent a structured series of intended learning outcomes for mathematics. But the anticipated learning outcomes described by the five objectives are all in a cognitive domain. To be a more complete curriculum for use in this study, anticipated learning outcomes in an affective domain, such as the students being expected to develop enjoyment and interest in learning, were added.

The first purpose of this study, to construct an instructional unit, can be viewed as an attempt to develop an instructional unit using inferential statistics as the instrumental content to serve the curriculum defined above. Specifically, the instructional unit in basic inferential statistics developed in this study is intended to facilitate the learning outcomes as described by the defined curriculum.

The second purpose of this study, to determine the feasibility of using the instructional unit, can be viewed as the actual instructional operation of using the developed unit in basic inferential statistics to serve the defined curriculum, and also the evaluation of that operation using the criteria based on the learning outcomes described by the curriculum. Since the actual instructional operations of the unit in basic inferential statistics were carried out by the teachers, the evaluation of the operations was also based on the teachers' opinions which involved both the learning outcomes and other aspects of the instructional unit which appeared in the Teacher Reaction Test.

The third purpose of this study, to investigate the students' anchoring ideas before and after the instruction of the unit, can also be viewed as related to the defined curriculum. The investigation of the students' pre anchoring ideas about statistics can be viewed as the investigation of the students' repertoire of statistical knowledge before going to the actual instructional operation of the unit in basic inferential statistics. In evaluating the instructional unit, the investigation of the students' post anchoring ideas paralleled one of the anticipated learning outcomes of the curriculum, the first objective of junior high mathematics—to develop an understanding of mathematical concepts and structures. However, the investigation of the students post anchoring ideas can also be viewed as the investigation of the change in the students' anchoring ideas which was brought about by the quality of the students' pre anchoring ideas and the presentation of the potentially meaningful materials—the instructional unit in basic inferential statistics.

Unit Constructibility

It was evident that the construction of the instructional unit depended on one's justification of what purposes it was expected the constructed unit would serve. The primary purpose of constructing the instructional unit employed in this study was to illustrate a basic structure of inferential statistics to grade nine students, anticipating that it would fulfill the objectives of junior high school mathematics of Alberta. Various underlying statistical concepts were considered and selected to form a defined structure of inferential statistics. The intuitive approach, along with the instructional strategy of using examples, experiments, and graphical illustrations, was selected as the communicating procedure for the unit.

The statistical content in the unit was selected on the basis of three sources of concern: psychological, social, and mathematical. Psychologically, the unit was intended to illustrate a basic structure of inferential statistics—the underlying statistical concepts and principles, and how they are related to form such a structure. According to Bruner (1960), the learning of a structure is very important because it serves in the future by rendering a so called "non specific transfer" or later efficient applicability to a new situation. Socially, the statistical content in the unit was selected on the basis of its usefulness for an ordinary citizen, as recommended to be included in the secondary school curriculum by various influential reports such as the Cambridge Conference and the College

Entrance Examination Board. Mathematically, the unit is intended to illustrate mathematical methods applying to statistics, such as the concept of score transformation.

The unit was also constructed in such a way that the students could intuitively learn the fundamental structure of inferential statistics. The unit was intended to be a first statistics course for non-formal-probability-background students. The unit is not a rigorous mathematics course. Therefore, with the proper communicating procedure—the intuitive approach and the modifications of the instructional materials to suit grade nine students' intellectual development as suggested by Bruner—the concepts and principles in the unit can be adequately learned by the students.

It is assumed that the students intuitively have an informal idea of probability as a prerequisite for studying the unit. It is not, then, necessary for a statistics course to be tied in as the application part of a probability course, as is the widespread belief in curriculum development. The National Advisory Committee on Mathematics Education (1975) points out that statistics has its own special features that can be presented along with mathematical method, not necessarily waiting to present it as an applied part of probability. Moreover, the tying of statistics to the formal probability concepts may be too rigorous and not suitable for general students. However, the unit is not isolated and complete in itself. It is considered a part of the continual process in teaching probability and statistics at the level of elementary and secondary school. It will do students no harm to have already had a formal course in probability, rather, it should help them learn more efficiently. On the other hand, what the students learn from the unit should facilitate their future study of more formal and rigorous inferential statistics courses.

The unit constructibility was justified with respect to its appropriateness for grade nine students, and the fulfillment of the five objectives of junior high school mathematics. From the researcher's point of view, the procedure of constructing the unit, which included the content analysis, instructional analysis and pilot study, aimed at the justification of the unit appropriateness for grade nine students by insuring that they can adequately learn and develop good attitudes toward the material in the unit. The empirical evidence for the unit constructibility with respect to its appropriateness for grade nine students indicates that grade nine students adequately learned the statistical concepts and principles with respect to each of the seven specific objectives of the unit. And, after studying the unit, they expressed the opinion that the instructional unit was not difficult for them.

The unit constructibility with respect to its appropriateness for the fulfillment of the five mathematics objectives was justified by the fact that the unit in basic inferential statistics can provide adequate instructional materials to fulfill each of the five objectives. For instance, the unit can provide instructional materials for topics such as mean, standard deviation and score transformation which require the students to practice their skill in the use of fundamental processes to fulfill the second objective of the five mathematics objectives. The empirical evidence collected for the unit feasibility assessment with respect to the five mathematics objectives was also used in considering the unit appropriateness.

In summary, the unit constructibility was considered according to its appropriateness for grade nine students and the five objectives of junior high school mathematics. First, its appropriateness was justified logically; then it was confirmed by empirical evidence from the unit feasibility assessment.

The Fulfillment of the Five Objectives of Mathematics

The instruction of a unit in basic inferential statistics was generally feasible with respect to the five objectives of junior high school mathematics in Alberta

For the first objective of junior high school mathematics, the results indicated that the students adequately learned statistical concepts and structures from the unit in basic inferential statistics. These results supported the findings of previous studies such as those of Shulte (1970), Smith (1966), and others. Moreover, the results lend support to the statement of Bruner (1960), "Any subject can be taught to any child at any stage in an intellectually honest form."

Although the feasibility with respect to this objective was subjectively judged by criterion referenced level setting, it could also be assessed by investigating the average scores achieved by each and both schools participating. They were: 67.93% for School A, 66.02% for School B, and 66.91% for both schools, indicating that the students' performance on the Achievement Test was high enough to judge the students' learning as adequate. There was not very much difference between the average scores of the two schools, even though the students in Sample A were considered to have higher ability than those in Sample B according to the descriptions of the samples in this study. However, the students in Sample B had four more study periods than those in Sample A.

For the second objective, the development of skill in the use of fundamental processes, the results indicated that the students' skill in the use of the four fundamental operations combined was significantly improved after the unit instruction. Separate analysis of individual operations indicated the significant improvement of skill in the use of subtraction, multiplication, and division, but no significant improvement for addition.

The improvement of skill in the use of multiplication but no significant improvement of skill with respect to addition in this study was consistent with the findings of the study by Shulte (1970), while the results with respect to subtraction and division were contrary to those of Shulte (1970) which indicated no significant improvement of skill in subtraction and decline of skill in division.

However, in this study the results of the skill tests for the two schools did not agree, with the exception of addition and division for which both schools indicated no significant and significant improvements respectively. For subtraction, multiplication, and the four operations combined, School A indicated significant improvement, while School B did not. This difference in results between the two schools may be attributed to one or a combination of the following reasons.

Generally, Sample A consisted of more high ability students than Sample B as illustrated by the fact that Sample A had a higher mean I.Q. score and higher averages for previous mathematics grades than Sample B. The rate of improvement of skill in using the four fundamental operations may be in favor of the high ability students.

Grade nine students do not have a formal course to practice skill in the use of fundamental processes, however, their skills are expected to improve with the time and experiences involved in any mathematics course. Sample B took the skill pretest about three weeks after Sample A and during those three weeks the students in Sample B studied their regular mathematics course which may have helped their skills. As evidence of this, Sample B had an average pretest score of 35, which was higher than that of Sample A, 33. Sample B wrote the skill post-test right after completing their two hour regular school mathematics examination. Many students looked tired and did not want to take the skill test at that time. This may have resulted in poorer performance than usual in writing the skill post-test, and consequently, in lower scores than those of Sample A. And finally, the significant improvements were not found in Sample B because of starting with higher scores on the pretest and finishing with lower scores on the post-test than Sample A.

The unit instruction was also feasible in regard to the third objective of mathematics, to develop systematic methods of problem solving. The results indicated that 62.22% of the students reached the criterion referenced score for the problem solving methods test. This illustrates that the students who studied the unit did adequately develop problem solving methods based on statistical ways of thinking. With respect to the open search problems in statistics, the students exhibited adequate problem solving methods equivalent to the three-stage problem solving method in statistics such as: defining selected samples, defining statistical techniques employed, and defining the statement of inference for the intended population.

In this study, the feasibility assessment with respect to this objective of mathematics tends to agree with the findings of the study by Mayer, Stiehl and Greeno (1975) which concluded that:

> In general, instruction that emphasizes meanings of concepts leads to knowledge that is more easily applied in new contexts, and is the basis for more flexible and interpretive use by the subject (p. 349).

The instruction of the unit in this study also emphasized the meanings of the concepts included. As a result, the students adequately developed more flexible and interpretive use of those concepts in establishing problem solving methods.

Furthermore, the students who expressed adequate problem solving methods with respect to the open search statistical problems were generally also those who adequately learned statistical concepts from the unit. There was a high correlation between the two aspects of learning. The computed correlation coefficient of 0.85 was found between the students' scores on the Achievement Test Part I on statistical concepts and the Achievement Test Part II on problem solving methods. This seems to indicate that students who adequately learn statistical concepts can also establish suitable problem solving methods based on the relationship of those statistical concepts.

The instruction of a unit in inferential statistics was feasible with respect to the fulfillment of the fourth objective of mathematics, to develop habits of precise thought and expression. From the results of this study we see that the students' observable behavior and expressions developed considerably toward habits of precise thought and expression. By its nature, this objective of mathematics is not testable. Developing habits involves the long term process of behaving in a certain way until it becomes relatively fixed in character and almost automatic in performance. Because the instruction of the unit took a short period of time, it may not have provided enough training for the students to develop habits of precise thought and expression involving statistics. However, the unit in inferential statistics, like any other unit in mathematics, has content which can be instrumental in training students in habits of precise thought and expression. The instructional variable plays a major role in training such habits. To achieve the objective, one must take it into account during instructional planning and the execution of those plans.

The instruction of the unit in basic inferential statistics was feasible with respect to the fifth objective of mathematics, to develop an understanding of the application of mathematics. The results indicated that 66.67% of the students reached the criterion referenced score for the test dealing with application. This evidence shows that the students adequately developed an understanding of how to apply statistical concepts to various unfamiliar situations.

In part, this aspect of the feasibility assessment tends to support the findings of the study by Mayer, Stiehl, and Greeno (1975) which concluded that:

> . . . Instruction that emphasizes meanings of concepts leads to knowledge that is more easily applied in new contexts (p. 349).

As already mentioned in the discussion of the results with respect to the third objective of mathematics, this unit did emphasize the meanings of concepts which resulted in the fulfillment of both objectives—to develop problem solving methods and to develop an understanding of application.

Statistics, by its nature, is applied mathematics. The teaching of statistics should involve the illustration of how to apply the learned concepts to various situations at various stages so that students can meaningfully learn those concepts and appreciate their usefulness. Furthermore, to achieve the objective of understanding how to apply the concepts, the problems involving application must be illustrated and taught along with the intended concepts. However, application problems usually tend to involve complicated manipulations and many distractions which may confuse students. Consideration must be given to the decision as to when to compromise efficiency of time and effort in order to include such problems.

In summary, the conclusions of the feasibility assessment for teaching the unit in inferential statistics with respect to the five objectives of junior high mathematics were as follows:

1. The students adequately learned statistical concepts and principles from the unit.

2. The instruction of the unit in basic inferential statistics resulted in the improvement of the students' skill in the use of fundamental process, all four fundamental arithmetical operations combined. Specifically, the students' skill in the use of each of the three operations: subtraction, multiplication, and division was significantly improved, but no significant improvement was evidenced with respect to addition.

3. The instruction of the unit in basic inferential statistics resulted in the students adequately developing problem solving methods based on statistical ways of thinking.

4. The instruction of the unit in basic inferential statistics provided the students with many situations which helped to develop habits of precise thought and expression.

5. The instruction of the unit in basic inferential statistics resulted in the students adequately developing an understanding of how to apply statistical concepts to unfamiliar situations.

Student Reactions to the Unit

The students' overall reactions were in favor of the unit with respect to twenty five opinionaire items categorized under five variables: enjoyment of activities, difficulty of the subject matter, interest of the content, feeling of new knowledge gained, and perception of the usefulness of statistics. However, the results for individual variables were not entirely favorable—the categories of enjoyment and interest indicated nonsignificant evidence in favor of the unit, while the other three variables were significantly in favor of the unit.

The results seem to illustrate that the students' reactions with respect to the two variables, enjoyment and interest, were more difficult to change than with respect to the other three variables, difficulty, new knowledge gained, and usefulness. Enjoyment and interest may involve deep-seated previous attitudes about mathematics which tend not to be appreciably changed by the instruction of any unit in a short period of time. The other three variables, difficulty, feeling of new knowledge gained, and usefulness involve attitudes which more directly relate to the students' ability to learn statistical concepts from the unit. In evidence of this, the students' performance on the Achievement Test indicated that they adequately learned statistical concepts from the unit which was consistent with their reaction that the unit was not difficult and that they had a feeling of new knowledge gained. And furthermore, the students' reaction that they perceived the usefulness of statistics was also consistent with the result of the test, indicating that they adequately developed an understanding of how to apply statistical concepts.

The students in School A and School B reacted relatively in agreement with respect to the two variables, difficulty and new knowledge gained, but differently with respect to the other three. The students in Sample A indicated that they enjoyed the activities and perceived the usefulness of statistics, but were nonsignificantly interested in the content. However, the result on the interest variable was significant at the .10 level and the mean of five-point scale responses was 3.19 which was considerably higher than the theoretical average of 3.

The students in Sample B, on the other hand, indicated that they did not enjoy the activities, were not interested in the content, and nonsignificantly perceived the usefulness of statistics. The student reactions with respect to enjoyment and interest from Sample B tended to support the finding of Shulte (1970) that the instruction of a unit in probability and statistics caused the students' attitudes to decline.

The incongruent results between Sample A and Sample B in this study seemed to be attributed mostly to the function of instruction and not very much to the content since the students in the two schools did not express ideas very much different with respect to the interest of the content, that is, Sample A showed no significant interest in the content while Sample B reacted with significant negative interest in the content. But the students in both samples reacted significantly in opposite directions with respect to the enjoyment of the activities.

As a part of instruction, the students in Sample A received different preparation. They started by studying graphs intended for use in the unit. They were taught slowly at the beginning and were given an opportunity to paint the graphical figures in various colors which provided enjoyment in addition to illustrating the intended statistical concepts. Then their work was displayed on the bulletin board in the classroom prepared for that purpose. The students were proud of their work and enjoyed the activities. This seemed to set the pace for good attitudes in studying statistics in the unit. The students in Sample B, on the other hand, were assessed as already having enough preparation in reading and constructing graphs. They started the lessons in the unit immediately which caused some of them to exhibit an uneasiness about studying statistics. Furthermore, from the classroom observations of the author, the instruction employed in Sample B tended to emphasize the definitions and formulas by having the students memorize those formal written statements instead of having them intuitively grasp the concepts. The students may have felt forced to do what they usually dislike in mathematics.

The different backgrounds of the students in the two samples may also combinatorily contribute to their different reactions to the unit instruction. Two thirds of the students in Sample A were described as average and high ability students. The I.Q. score average was 107.43 and the mathematics grade average for the previous term was 71.19%. The students in Sample B were quite opposite, about two thirds were described as average and low ability students. The I.Q. score mean was 101.11 and the mathematics grade average in the previous term was 52.59%.

Another factor that may also have contributed to the difference in the student reactions between the two schools was the procedure employed in forming "the option classes" which were selected as Sample A and Sample B in this study. In Sample A, the students selected the math option class according to their own interest, while in Sample B, the students had a choice between French and Mathematics which were offered at the same time as option classes. The students might not have been interested in the math option, but they did not have much choice because a lot of them did not like French either, according to the teacher. The willingness of the students to study the unit in basic inferential statistics seemed to vary between the two samples from the beginning and may have resulted in the different reactions to the unit instruction as described.

Student comments generally were agreeable with the results of the questionnaire, in particular, with respect to the two variables, enjoyment and interest. Almost all fifteen comments made by the students in Sample A were in favor of the unit. As one student in Sample A put it:

> I really like the course. It taught me something that I did not know and helped me to understand how statistics can be used and ran in everyday life.

For Sample B, most of the seventeen comments made by the students were not in favor of the unit. However, the majority of those neutralized their comments by pointing out both some particular likes and dislikes. As one student in Sample B said:

> I did enjoy doing the new class. But as any new thing to accomplish, it was somewhat strange and difficult.

In conclusion, the student reactions toward the instruction of the unit in basic inferential statistics were as follows:

1. There was no significant evidence from the students' questionnaire that they enjoyed the activities in the unit.

2. The students expressed the opinion that the unit was not difficult.

3. There was no significant evidence from the questionnaire that the students were interested in the content of the unit.

4. The students expressed the feeling that they had gained a lot of new knowledge from studying the unit.

5. The students expressed the opinion that they perceived the usefulness of statistics from studying the unit.

6. The students expressed the overall opinion (with respect to the combination of the five variables: enjoyment, difficulty, interest, new knowledge gained, and usefulness) that they were in favor of the unit.

Teacher Reactions to the Unit

The two teachers who taught the unit generally expressed positive opinions toward the instruction of a unit in inferential statistics for grade nine students. They believed that the unit instruction would fulfill each of the five objectives of junior high mathematics in varying degrees. They both agreed that the students enjoyed studying the unit, especially the teacher in School A who stressed this point by commenting that:

> Interest was great. Students did work on their own time at home, some stayed after school hours and questioned me about various areas taken.

The teachers felt that they themselves had sufficient subject matter background and so would general junior high mathematics teachers. The instructional suitability of the unit was satisfactory according to the teachers. However, one commented on not having enough teaching time and the other would like to see the materials for the students more complete so that students could work individually and also would like to see more experiments involving the concepts of certain lessons, such as variability—standard deviation. The teachers felt that this type of mathematics could be included in the regular school mathematics program if time allotment was made available.

The Students' Anchoring Ideas

The investigation of the students' anchoring ideas before and after the instruction of the unit in basic inferential statistics, and also the analysis of the change between the pre and post anchoring ideas, yielded the following conclusions:

1. Grade nine students had some good quality anchoring ideas with respect to some selected statistical concepts before those concepts were taught in the regular classroom. However, their ideas with respect to a certain statistical concept were in varying degrees of quality depending on the degree of difficulty of that selected concept.

Specifically, with respect to the eight selected statistical concepts ((1) Measures of central tendency, (2) Variability of scores, (3) The effect on a mean when a constant is added to the data, (4) The effect on a mean when the data are multiplied by a constant, (5) The effect on a variance when a constant is added to the data, (6) A normal distribution, (7) The use of a sample mean as the estimate of a population mean, and (8) The effect of using a mean from a different size sample to estimate a population mean), at least some students' ideas among several different ideas expressed with respect to each of those concepts were considered to be the correct ideas according to the nature of the statistical concepts tested. Among the eight selected concepts, only on three concepts (1, 6, and 8) did the majority of students express correct ideas in the pretest. The two most difficult concepts for the students were concept 2 and 5 for which a very small number of correct ideas were expressed by the students.

2. The students' pre anchoring ideas tended to be more scattered, having more different ideas than on the post-test with respect to the same statistical concept. The students' post anchoring ideas on a particular statistical concept, on the other hand, tended to be more homogeneous and of better quality. There were more different pre anchoring ideas than different post anchoring ideas with respect to almost all eight selected statistical concepts. And for each of those concepts, there were more quality ideas appearing in the post-test than in the pretest.

3. The students who held quality pre anchoring ideas tended also to hold equal or better quality post anchoring ideas with respect to the same statistical concept. It was evident from the study that most students who held pre anchoring ideas at the level of at least S^-, or the lowest level of the "satisfactory" category of answers, tended also to hold at least the same quality post anchoring ideas with respect to almost all statistical concepts tested.

On the other hand, the students who did not hold good quality pre anchoring ideas tended also not to hold good quality post anchoring ideas with respect to some statistical concepts tested. For example, with respect to the concept of variability, 59.52% of the students expressed pre anchoring ideas categorized as "U", or "unsatisfactory"; and from the same group 52.38% still expressed post anchoring ideas at the unsatisfactory level. However, with respect to other statistical concepts tested, such as measures of central tendency, and the transformation of a mean when data are added to or multiplied by a constant, there were a fair number of students who held unsatisfactory pre anchoring ideas but changed to satisfactory post anchoring ideas later. This incidence seems to indicate that the students' designated unsatisfactory pre anchoring ideas do not necessarily mean that they are not relevant to the students' learning. It depends on the nature of the concept to be learned. And furthermore, the students may intuitively have had relevant pre anchoring ideas with respect to the concepts tested but failed to express precise ideas; consequently their pre anchoring ideas were judged as "unsatisfactory."

This aspect of the conclusions seems to support Ausubel's statement that what a learner already knows essentially determines the learning outcome. The good quality pre anchoring ideas can be viewed as the students' relevant background to a particular statistical concept and the better or equal quality post anchoring ideas with respect to the same statistical concept as the students' learning outcomes. Having equal or better ideas as a learning outcome is due partly to the existence of relevant background.

4. The instructional unit in basic inferential statistics caused the students' original anchoring ideas about certain statistical concepts to change into better anchoring ideas with respect to the same statistical concepts.

From Table 7.1 we see that there were more students who changed their pre anchoring ideas into better quality post anchoring ideas than those who changed in the opposite direction with respect to almost all selected statistical concepts. On two selected statistical concepts, namely, Concept 5—the transformation of a variance, and Concept 6—a normal distribution, the instruction of the unit did not result in better quality anchoring ideas.

The concept of the transformation of a variance when a constant is added to the data seemed to be difficult for the students. In the pretest half of the total number of the students suggested computing the new variance from the new set of data transformed. In the post-test only about 10% of the students suggested the same idea. Instead, 45% of the students suggested using the transforming idea by adding the constant to the original variance to get the new one. This idea illustrated by the students, however, was not correct according to the nature of the concept. It was therefore classified as a lower quality idea than the idea of suggesting working it out from the new set of data since the latter idea would eventually proceed to the correct answer, a new variance, although without using the idea of transformation. As a result, about 21% of the students shifted their ideas from "suggesting to work it out" to "using incorrect transforming idea" between pre- and post-tests; this one incidence contributed in a major way to the test results indicating that the students' anchoring ideas with respect to this concept declined in quality as a result of the unit instruction. Moreover, about 12% of the students did not give an answer on the concept in the post-test, and those students gave higher quality anchoring ideas in the pretest. Accordingly, a large decline in the quality of the students' anchoring ideas with respect to this concept was shown.

To change ideas from a suggestion to work it out directly from the new set to using an incorrect transforming idea may be due to the students' impressions from using the suggested transforming idea for the problem about the transformation of a mean when a constant is added to the data. The students seemed to conclude that once a set of data is transformed to another set by a certain arithmetic operation, its characteristics like mean, standard deviation, variance, etc. can be transformed to those of the new set by the same operation. It was logical thinking for them. Furthermore, besides the difficulty and abstractness of the concept of a variance as a measure of data variability, the concept of variance was not emphasized in the unit. It appears as one prior step in calculating the standard deviation.

In addition, because of limited instructional time, the last five lessons of the unit were hastily taught, as illustrated in Tables 5.2, and 5.3 (Day-by-Day Instructional Plan). Among those five lessons were included two lessons, 10 and 11, dealing with the concept of transformation. Particularly, the students in School A did not finish the last part of Lesson 10 dealing with the transformation of a standard deviation and a variance; moreover, they skipped Lesson 11, which provides the activities or exercises involving the concept of transformation, completely.

Finally, a number of students did not express any ideas in the post-test but simply left a blank space or wrote "I don't know." These types of answers were listed as lowest rank and categorized as unsatisfactory. This high incidence of failure to respond may be due to the way the post-test was administered. Because of limited time, all the students in the whole class were instructed to take the test at the same time, instead of partially interviewing the students individually or in a small group as was done in the pretest. As a result, it was beyond the control of the test supervisor to have each student complete every single test item on those selected statistical concepts.

However, the unit instruction did cause a number of students' anchoring ideas to change into better quality ones with respect to the concept of the transformation of a variance. About 21% of the students expressed correct ideas on the concept in the post-test compared to 5% in the pretest.

The concept of a normal distribution, one of the areas of decline in quality of anchoring ideas, was measured by asking the students to give examples of the distribution after one example was given as an illustration. The examples given in the test involved the illustration of students' heights which naturally form a normal distribution. The students' pre anchoring ideas with respect to this concept mostly involved students' weights which also naturally form a normal distribution. The ideas expressed by the students may have been influenced by the use of heights in the given example. As a result, the majority of the students' pre anchoring ideas were considered as high quality ideas categorized under "the natural phenomena of a normal distribution" because their ideas involved continuous data such as height, weight, etc. which naturally possess an approximately normal distribution.

The students' post anchoring ideas with respect to the concept of a normal distribution were more independent, rather than imitations of the given example in the test. The students freely established a variety of examples of a normal distribution according to their own understanding. As a result, 91 different examples were given by the students in the post-test compared to 60 examples given in the pre-test to illustrate their anchoring ideas with respect to the concept of a normal distribution. However, a number of those examples given by the students in the post-test involved discrete data which is considered to be approximately a normal distribution with specific justification, and categorized as lower quality ideas than the natural phenomenon of a normal distribution.

Consequently, the effect of the unit instruction was analyzed as causing the students' anchoring ideas on the concept of a normal distribution to change to lower quality anchoring ideas. Actually, the students may not be aware of the difference between discrete and continuous data, because these concepts were not taught in the unit. Even though the unit disregarded the difference between the use of continuous data categorized as the natural phenomenon of a normal distribution and discrete data categorized as being a normal distribution only in specific justified cases, the unit instruction did give the students a greater understanding of the concept. They created a larger variety of different ideas in the post-test than in the pretest, most of which were considered examples independent from the influence of the given example in the test. Furthermore, more examples involving the use of continuous data were expressed in the post-test than in the pretest, however, a smaller number of students in the post-test were classified as having this category of idea because only the student who expressed a majority of examples, from his several examples, involving the use of continuous data was classified as being in this category.

Another statistical concept, variability of scores, was difficult for the students to understand. Only 14.29% of the students changed their anchoring ideas to better quality ideas with respect to this concept compared to 9.52% who changed in the opposite direction. The majority of the students, 76.19%, did not change their ideas, and among those, 52.38% remained in the category of "U" (Unsatisfactory).

The above incidence may be due to the fact that the concept of variability is difficult and abstract beyond the students' intuitive experience. The students may also have had some difficulty in expressing their ideas about the concept. As evidence of this possibility, the majority of students in both pre- and post-tests indicated that they perceived the differences among the three given sets of scores but failed to explain the differences in relation to the concept of variability (see Table 6.12).

Generally, the instructional unit in basic inferential statistics did cause the students' anchoring ideas about statistical concepts to change to better quality ideas. According to Ausubel's meaningful learning theory, the students' post anchoring ideas can be viewed as learning outcomes, the lessons in the unit as the potentially meaningful learning materials, and the students' pre anchoring ideas as the background to which the learning materials are related in order to achieve the outcome. The nature of the learning materials, which includes the nature of statistical concepts used and the nature of the students' pre anchoring ideas or relevant background experienced outside the course, both combinatorily contribute to the learning outcome, or the nature of the students' post anchoring ideas.

In summary, the conclusions with respect to the investigation of the students' anchoring ideas about statistical concepts were:

1. The students seemed to have some ideas about statistical concepts before studying the unit in basic inferential statistics.

2. The students' ideas about statistical concepts tended to be more scattered before studying the unit, then tended to be more homogenous and of better quality after the completion of the unit instruction.

3. The students who seemed to have some ideas about statistical concepts before studying the unit, also held equal or better quality ideas after the completion of the unit instruction.

4. The instructional unit in basic inferential statistics caused the students' ideas about statistical concepts to be better in quality.

Limitations of the Study

The interpretation of the results of this study should be made with regard for the following limitations.

1. In this study a set of selected statistical concepts was used to form an instructional unit in basic inferential statistics. The results of this study may not be appropriate for other content not covered in the unit or for the whole content population of inferential statistics.

2. The subjects who participated in the study were not randomly selected. Instead, the option classes formed by the schools involved were assigned to study the unit. The generalization of the results with respect to a grade nine population should be interpreted according to the extent of such limitations as described in the samples.

3. The subjects who participated in the study also studied their regular mathematics course, algebra, during the same period of time as they were studying the unit in basic inferential statistics. This uncontrolled aspect may have combinatorily contributed to the results of the study, in particular, the feasibility assessment with respect to the students' skill in the use of fundamental processes. Because grade nine students do not take a formal course particularly for the purpose of practicing skill in the use of the fundamental process, and their skill could be affected by any mathematics course, this aspect of study may be generalized with regard for such limitations.

4. Due to limited time for testing, the administration of the Conception Test was modified. For the pretest, the partial interview procedure was used to test the students individually or in a small group of five, but for the post-test, the students in a whole class were instructed to write the test at the same time. Some students may have chosen not to answer some questions or to opt out on some points in clarifying their ideas because the test supervisor had too many students to supervise at the same time. The results of this study with respect to the students' anchoring ideas should be

regarded in the light of such limitations.

RECOMMENDATIONS

A. Inclusion of a Unit in Inferential Statistics in the Regular School Mathematics Program

The author strongly recommends the inclusion of an instructional unit in basic inferential statistics in the regular school mathematics program, in particular for grade nine, on the basis of the feasibility of its fulfilling the five objectives of junior high mathematics, and the overall positive reactions to the unit from both the students and the teachers. Furthermore, the instructional unit in basic inferential statistics also helped the students develop better quality anchoring ideas about statistical concepts. This should be the ultimate goal of introducing probability and statistics to the students, that is, to influence their thoughts or ideas about various situations in society toward the statistical way of thinking.

B. Development of an Instructional Unit

Based on the experience in developing the unit in basic inferential statistics employed in this study, the following recommendations are made for a similar development of an instructional unit in statistics.

1. An instructional unit should be constructed based on the structure of the subject matter and a structure of various underlying concepts for that subject matter. In other words, the construction of an instructional unit should be related to the over-all structure of the subject matter which it is intended the learner grasp, and

the subsequent related structures of various underlying concepts that form the whole structure. In the instructional unit employed in this study, the whole structure of basic inferential statistics is the relationship of the three components—a population, a sample, and an attempt to get information of a population by using information from a random sample. The subsequent structures of various statistical concepts, such as various descriptive characteristics for a population and a sample, and statistical estimation, are then included.

Generally, a structure intended to be included in a unit is investigated and simplified first. Then various basic structures of concepts underlying the intended structure are chosen to form a unit. The meanings of the concepts and the relationships among various selected concepts intended to form a structure are important, and should be emphasized as selecting criteria. In summary, a structure of the subject should be overviewed and analyzed first; then details of various basic concepts can be added.

2. Make use of the students' intuition in developing a communicating procedure, or instructional strategy. The intuitive approach is recommended and it is worthwhile to start using it at any level of study. The illustration of concepts by using appropriate created examples or experiments is also satisfactory.

3. Make use of the students' curiosity about situations involving themselves or related to them. The use of students' personal data motivates learning. Meaningful real data involving their environment also interest them. These types of data are recommended for use in an instructional unit.

4. Do not overemphasize symbols, formulas, computational techniques, and manipulation of those symbols and formulas in developing an instructional unit in statistics. Instead, emphasis should be put on the meanings of concepts and the usefulness of the subject, because most students get bogged down by the overemphasis of those symbols, formulas, and computational techniques without grasping the actual ideas of statistics which is the ultimate goal of teaching statistics. Moreover, those students develop negative attitudes toward statistics as involving too complicated mathematical manipulations.

C. Use and Improvement of Materials in the Unit

The following recommendations about the use and improvement of materials in the unit in basic inferential statistics are made on the basis of the experience gained during the unit instruction.

1. Care should be taken to introduce the unit before beginning the actual lessons, making sure that the students' backgrounds meet the prerequisites for the unit, motivating the students' interest and building up their confidence. Evidently, the students in School A of this study had a better introduction, setting the pace for studying the unit. As a result, the students reacted favorably to the unit instruction.

Specifically, the students' skill in reading and constructing various graphs should be assured. More exercises could be added to Lessons 1 and 2 to help the students practice and develop these skills.

2. Although the instructional time for the unit was intended

to be about 20 forty-minute periods, the actual teaching time should be flexible depending on the degree of the students' understanding. A number of students commented that the course was too rushed near the end of the unit. This seems to indicate that the students needed more than 20 forty-minute periods in order to feel they had a good grasp of the statistical concepts.

3. The students enjoyed the activities of decorating and painting the illustrated graphical figures and then displaying their work as is evident from the enjoyment of the students in School A. The amount of these types of activities could be considerably increased in the unit instruction. However, the intended statistical concepts behind those illustrations should be made clear to the students—which is the ultimate goal of the presentation.

4. The primary structure of statistics the students are intended to grasp in this unit is the basic inferential statistical procedure of using a sample to get information about a population. Therefore, the materials in this unit should be used to help the students understand and appreciate this structure.

5. There must be a compromise in the use of the materials in order to serve all the intended objectives of the unit. The unit was used in this study with the intention of serving the five objectives of junior high mathematics as well as teaching statistical concepts. Therefore, each of the five objectives had to be taken into consideration during the instructional process. For example, students had to do their own computations—aiming at serving the objective of developing skill in the use of fundamental processes.

Generally, the materials are not developed for only particular objectives. The major determinant of learning outcome is the instruction, or instructional strategy, and the learning outcome, as generally accepted, involves three domains: cognitive, affective, and psychomotor.

D. The Investigation of Anchoring Ideas

From the experience and the results of investigating the students' anchoring ideas about statistical concepts in this study, the following recommendations about usefulness, advantages and disadvantages for further development of a research procedure, are made.

1. The investigation of the students' pre anchoring ideas is useful in assessing the students' subject background for incorporation into the development of the instructional materials and the instruction of the subject matter. Some statistical concepts, such as variability of scores and transformation of variance, seemed to be difficult and too abstract for the students as indicated in the results of this study. Therefore, special care should be taken in developing instructional materials for these two concepts, and in teaching them. For instance, more concrete illustrative examples could be developed and more instructional time considered.

Some statistical concepts, on the other hand, did not seem to be very difficult or too abstract for the students. However, the results are still related to the quality of both instructional material developed and instruction. The results of this study indicated that the students already seemed to be familiar with the use of the concept of average as a measure of central tendency. Therefore, in developing the instructional material and actually presenting it, too much time should not be spent on this aspect. Emphasis should be placed rather on other aspects of the concept, such as median and mode as other measures of central tendency.

Specifically, an investigation of the students' anchoring ideas should be made prior to the construction of an instructional unit, since the results are useful for the development and the instruction as described.

2. The investigation of the students' post anchoring ideas is useful in assessing the type of learning the students are expected to accomplish. It is more significant than the Achievement Test because it involves more inclusive ideas which are expected to be the anchorage in cognitive structure for future use.

Ausubel emphasizes a meaningful learning which promotes clarity, stability, and retention of anchoring ideas for maximal relatability to a later learning situation. The investigation of anchoring ideas with respect to Ausubel's learning theory should aim at assessing the retention or longevity of the students' anchoring ideas, which partly implies the degree of clarity and stability.

Bruner, on the other hand, emphasizes the learning of a structure which promotes nonspecific transfer of learning for more efficient applicability in the future. The investigation of anchoring ideas with respect to Bruner's learning theory should involve assessing the applicability of a learned structure to various situations.

Specifically, the investigation of the students' post anchoring ideas should aim at the assessment of underlying learning outcomes which cannot be directly measured by achievement tests.

3. The testing procedure employed in this study to investigate the students' anchoring ideas is generally appropriate for the intended purpose. The development of a test which employs examples to illustrate the testing situations, followed by the intended questions, is recommended. The communicating procedure which employs a partial interview procedure is good, in particular, for the pretest. However, there is the disadvantage of too much testing time being required for the interview procedure.

One possible recommendation is to use more interviewers operating at the same time, each of whom follows the same instructions agreed upon. The other possible recommendation is to modify the Conception Test such that the students in the whole class can take it at the same time by constructing a test with open end choices of answers. The choices of answers may be developed with respect to the results of previous studies (if any), and a blank provided for each test item as an open end for adding different answers. The results of the investigation of the students' anchoring ideas from this study can be used as possible answering choices for the same selected statistical concepts.

E. For Further Study

With respect to the three purposes of this study to construct an instructional unit, to determine the feasibility of using the unit, and to investigate the students' anchoring ideas, the following further studies are suggested:

1. There exist several variations in developing an instructional unit in statistics. There is no absolute evidence indicating that one particular procedure in developing a unit is superior to the others in all aspects. There is always a place for a unit developer to create a new instructional unit with respect to one's own justification. One aspect seems to be common among the results of various studies involving the feasibility assessment of a constructed instructional unit, that is that, in the cognitive domain, students can adequately learn statistical concepts and principles in an instructional unit as indicated by such studies as those of Drake (1941), Smith (1966), Shulte (1970), and this study. However, the results with respect to the affective domain are not congruent, in particular, in relation to the students' deep-seated attitudes such as enjoyment and interest. Therefore, there exists a place for further study in developing an instructional unit aiming at improvement of the students' attitudes.

2. Further feasibility studies on the instruction of inferential statistics could be done in relation to the various aspects of the results of this study.

2.1 Since the study involved only 48 grade nine students, the materials in the unit could be used to teach a larger number of grade nine students in order to further substantiate the results. And since this study involved only the grade nine level, the materials in the unit could also be tried out in grade seven or eight as a feasibility study and to assist in judging the grade level most suitable for the material. The materials in the unit could also be used to teach different specified types of students in various grades so that the materials could be improved to suit those types of students since the ultimate goal is to provide all students with some statistical knowledge before going out into society.

2.2 The results of this study indicated that the instructional variable seems to be the major factor influencing the students' attitude toward the instruction of statistics. The materials in the unit of this study could be modified by using other types of instruction. For instance, the lessons employed in this study could be modified by using the individualized instruction method.

2.3 The statistical content covered by the unit employed in this study is only a set of selected concepts from the large field of inferential statistics, the highest topic covered is statistical estimation. An instructional unit in inferential statistics covering as high as the hypothesis testing concept could be developed and tried out in various grades.

2.4 Because statistics by its nature involves a lot of mathematical computation, an instructional unit in inferential statistics could be developed for use with a small electronic calculator or a computer. Many students during this study complained about the computation of standard deviation, some also indicated negative comments about standard deviation in the Student Reaction Test. Moreover, small electronic calculators are becoming commonly used by ordinary citizens in everyday life in modern society.

3. The investigation of the students' anchoring ideas about statistical concepts in this study can be viewed as a preliminary

exploration into this aspect of the study. Further study could be done to enlarge the field.

3.1 The investigation of the students' anchoring ideas could be further studied by involving more students and in various grades, so that the results of the study would be more conclusive and be significant evidence for later reference.

3.2 The testing instrument called the Conception Test in this study needs further modification to maximize the validity and reliability of the results, and to make it more convenient and effective for use involving a large number of students in various different types of statistical concepts.

3.3 The investigation of the students' anchoring ideas about problem solving methods based on statistical ways of thinking similar to those appearing in the Achievement Test Part II in this study is worth further exploration. How do the students change their anchoring ideas on problem solving methods, especially for the open search problems, during a unit of instruction in statistics? To develop a suitable problem solving method based on statistical principles is an important effect expected from statistics instruction.

3.4 The students' pre anchoring idea is the most influential factor determining the learning outcome according to Ausubel. Can the degree of relevance of a students' pre anchoring idea, or background, to a particular learning task involving statistics be assessed? This aspect is worth further investigation.

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APPENDICES

APPENDIX A

A TEACHER COPY: LESSONS ON BASIC INFERENTIAL STATISTICS FOR GRADE NINE

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LESSON 1 FREQUENCY DISTRIBUTION

PURPOSE: To introduce the basic concept of frequency distribution: A frequency distribution table.

EXPERIMENT: Suppose 30 grade nine students of a certain school have already been randomly sampled to measure their heights (in centimeters) with the following results:

162	166	163	164	167	165	168	166	164	165
163	169	166	164	167	165	168	166	167	166
166	170	169	164	167	165	168	166	168	167

MATERIAL:

PROCEDURE: Study and follow the illustrations:

Part A: A frequency distribution table

1. When we look at the data in our sample, we can not see clearly what heights the majority of students seem to have. The following table will help to figure it out.

Students' heights	Boundaries	Tally	f, number of students within these boundaries
170	169.5-170.5	1	1
169	168.5-169.5	11	2
168	167.5-168.5	1111	4
167	166.5-167.5	~~1111~~	5
166	165.5-166.5	~~1111~~ 11	7
165	164.5-165.5	1111	4
164	163.5-164.5	1111	4
163	162.5-163.5	11	2
162	161.5-162.5	1	1

Table 1

1.1 Can you tell the number of students for every height in the sample?

1.2 What height has the most number of students?

1.3 What height has the second most number of students?

1.4 What heights have the third most number of students?

1.5 If one would describe the sample that the majority of 30 students seem to have their heights between 164-168 cm. Would you agree?

2. From Table 1, you have learned:

2.1 Frequency

What is frequency?

What does the total frequency represent?

2.2 Class interval or simply called "interval" as the example 169.5-170.5, 164.5-165.5 etc.

2.3 Boundaries of an interval

There are two boundaries for each interval: the right most boundary or the upper boundary, and the left most boundary or the lower boundary.

Example: 167.5-168.5 is a class interval, the right most boundary is 168.5 and the left most boundary is 167.5

2.4 The width of an interval is simply the distance between the left most boundary to the right most boundary. The width of the interval 167.5-168.5 is 1. Does every interval in Table 1 have the same width?

2.5 Observe: the students' heights are arranged in order of magnitude, we call a set of ordered scores (x). For each score (x) there is a corresponding number of students (what is known as a frequency), we may call a set of corresponding frequency (f).

A set of ordered scores (x) together with a set of their corresponding frequencies is called a frequency distribution.

Example:

x	f
170	1
169	2
168	4
167	5
166	7
165	4
164	4
163	2
162	1
	$\Sigma f=30$

3. Sometimes the width of an interval, l, is too small to clearly see the important feature of the frequency distribution. We can enlarge the width of an interval as we want.

Complete the following tables:

Table 3A

Students' heights	Boundaries	Frequency (f)
169 or 170	......	......
167 or 168	......	......
165 or 166	......	......
163 or 164	......	......
161 or 162	160.5-162.5	1

Σf = ...

Table 3B

Students' heights	Boundaries	Frequency (f)
168 - 170	......	7
165 - 167	164.5-167.5	......
......	......	......

Σf = ...

3.1 What is the width of an interval in Table 3A?
in Table 3B?

3.2 From Table 3B the students' heights between 168-170 has a frequency of 7. Could you tell exactly what the frequency for each one of 168, 169, or 170 is, without checking back in Table 1 or the original data?

3.3 What heights have the most frequency in Table 3A?
in Table 3B?

3.4 Observe from Table 3A and Table 3B that we lose some detail about frequencies, especially for particular height. However, in some cases, we still need to divide the scores into big class intervals for clearly seeing the important features of the frequency distribution; for instance if there is a large number of scores or the difference between the smallest and the greatest score is very big.

LESSON 2 GRAPH OF FREQUENCY DISTRIBUTION

PURPOSE: To use graphs to illustrate the basic concepts of a frequency distribution

1. A histogram
2. A frequency polygon

EXPERIMENT: -

MATERIAL: Frequency distribution table from Lesson 1.

PROCEDURE: Study and follow the illustrations:

A histogram

1. Frequency distribution can also be graphically illustrated: A histogram can be constructed from a frequency distribution table.

Table 1 is used to construct the following histogram:

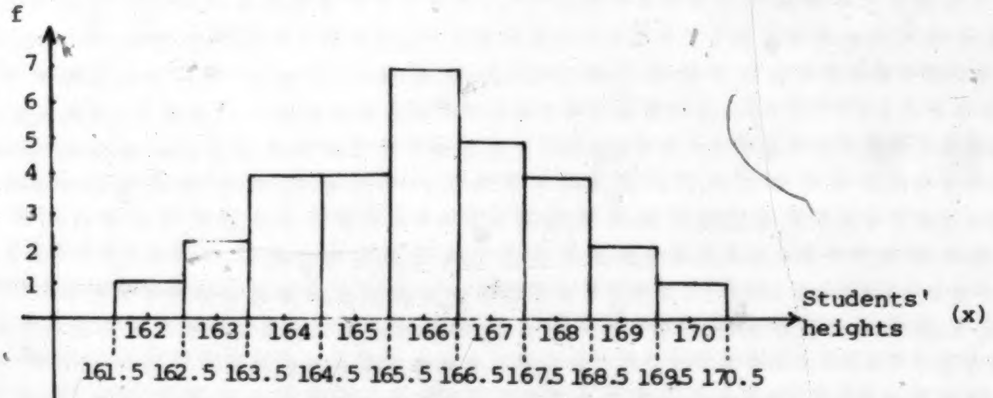


Figure 1

When a frequency distribution is illustrated by a histogram, a set of scores (the students' heights) and a set of their corresponding frequencies are designated by rectangular boxes.

1.1 What does a height of a box indicate?

1.2 What does a horizontal side of a box represent?

1.3 What is the total area of the above histogram?

1.4 What is the area of the box of the interval 165.5-166.5?

1.5 If you would like to randomly select one student from the above 30 students, what is the probability of getting a

166 centimeter height student?

1.6 What does the answer for Question 5 relate to the answers of Questions 3 and 4?

1.7 What is $P(x \leqslant 165)$? when x represents the students' heights?

1.8 Could you express $P(x \geqslant 167)$ in terms of the areas of boxes in the histogram?

2. A histogram can also be constructed by using a set of scores and their corresponding relative frequencies as follows:

Scores (x)	Frequency (f)	Relative frequency (f/n)
170	1	1/30
169	2	2/30
168	4	4/30
167	5	5/30
166	7	7/30
165	4	4/30
164	4	4/30
163	2	2/30
162	1	1/30
	N = 30	

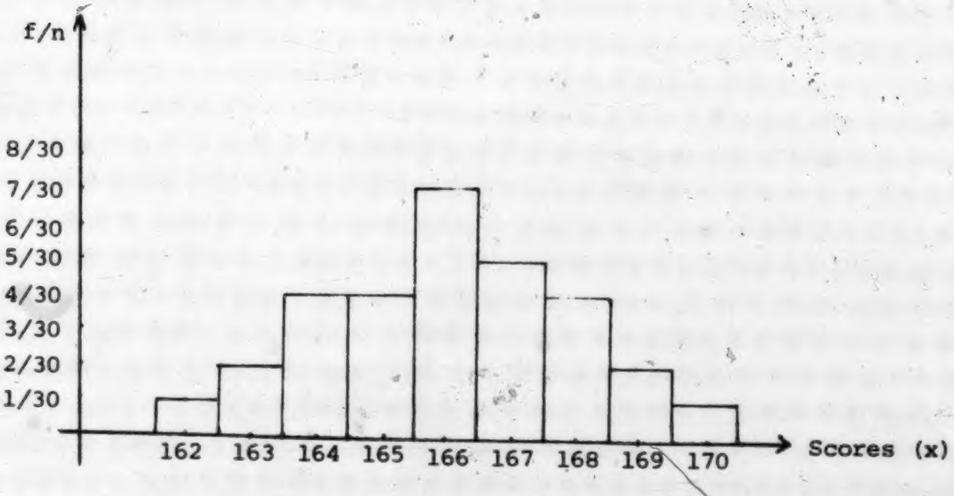


Figure 2

2.1 Do you observe what relative frequency is?

2.2 Does the histogram in Figure 2 look different from the one in Figure 1?

2.3 What does a height of a box indicate?

2.4 What is the total area of the histogram?

2.5 What is $P(x = 166)$? Is it the same as the area of box for 166 cm.?

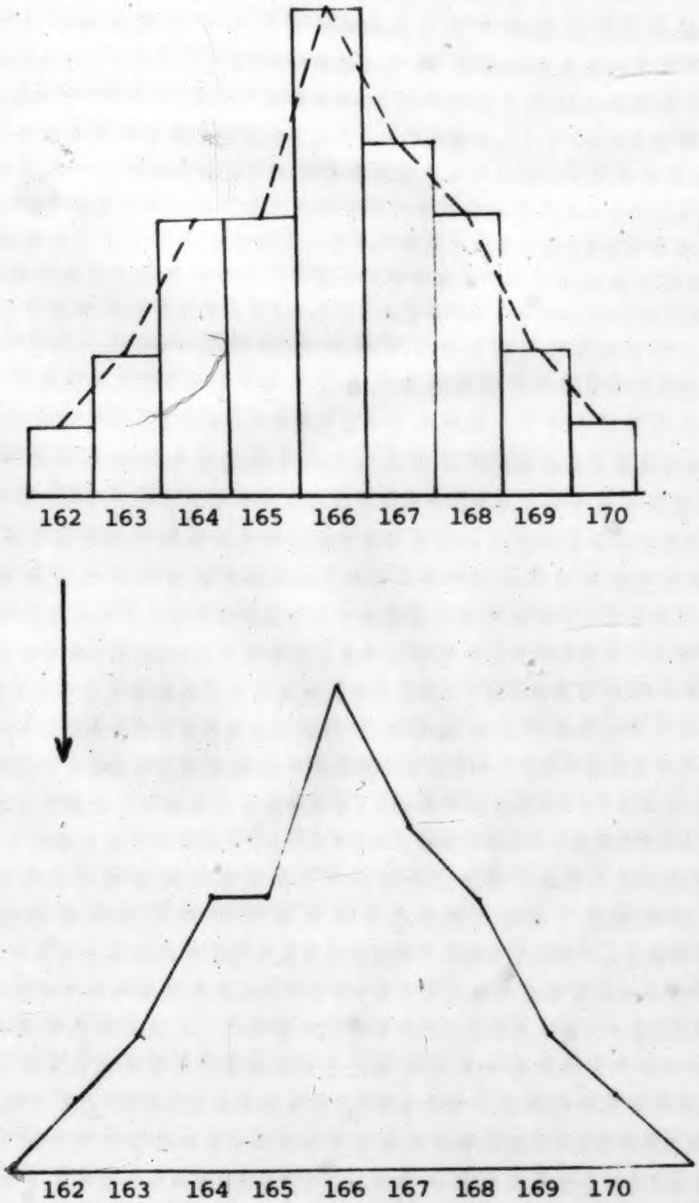
2.6 If you find the area from box 162 to box 165, what does the total area represent?

2.7 What is $P(x \geqslant 167)$?

2.8 If $P(162 \text{ to } x) = 1/2$; can you tell the value of x?

A Frequency Polygon

3. A frequency polygon can be constructed from a histogram. The histogram in Figure 4 is used to construct a frequency polygon as follows:



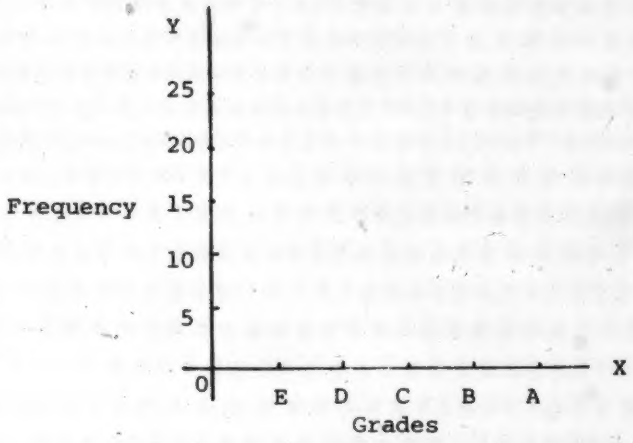
Frequency Polygon

4. A frequency polygon can also be constructed directly from a frequency distribution table.

Construct the frequency polygons from the following table:

4.1

Grades	Frequency
A	5
B	15
C	25
D	10
E	5



4.2

Scores	Midpoint	Frequency
16-18	17	1
13-15	14	3
10-12	11	5
7- 9	8	4
4- 6	5	2

5. Suppose the weights of 40 grade nine students have been measured in kilograms with the following results:

74, 73, 72, 70, 69, 69, 68, 68, 67, 67, 66,
66, 66, 65, 65, 65, 65, 64, 64, 63, 63, 63,
62, 62, 62, 61, 61, 60, 60, 59, 59, 58, 57,
57, 56, 55, 54, 52, 50

5.1 Complete the following table by using the above data.

x	Tally	Frequency
74		
73		
72		
71		
70		
69		
68		
67		
66		
65		
64		
63		
62		
61		
60		
59		
58		
57		
56		
55		
54		
53		
52		
51		
50		

5.2 Complete the following table by using the data in 5 or 5.1.

x	Midpoint	Boundaries	f
70 - 74	72		
65 - 69	67		
60 - 64	62		
55 - 59	57		
50 - 54	52		

$\Sigma f =$ ....

5.3 Construct the histogram for Table 5.1 and 5.2.
Which histogram seems to clearly indicate the important feature of the distribution of the students' weight?

LESSON 3 SHAPES OF FREQUENCY DISTRIBUTION

PURPOSE: To introduce the basic concepts of the following frequency distribution shapes:

1. Bell-shaped distribution
2. Rectangular distribution
3. U-shaped distribution
4. Triangular distribution
5. Miscellaneous distribution

EXPERIMENT: -

MATERIAL: -

PROCEDURE:
1. Draw a histogram for every example.
2. Observe if a histogram describes the name of a distribution.

The shapes of frequency distributions occurring in statistical practice vary considerably. The following examples will illustrate some of those frequency distributions.

1. Example of a bell-shaped distribution:

A group of students performs a certain physical education activity. A teacher uses a nine point scale to grade them as the following table:

Score scale	No. of students
1	1
2	3
3	5
4	10
5	15
6	10
7	5
8	3
9	1

2. Example of a rectangular distribution:

The following table is the distribution of the number of students in a certain elementary school.

Grade	No. of students
1	25
2	25
3	25
4	25
5	25
6	25

3. Example of a U-shaped distribution:

Twenty-one families are selected to investigate the family size. The result is as follows:

No. of people in a family	No. of family
3	5
4	3
5	2
6	1
7	2
8	3
9	5

4. Example of a triangular distribution:

The annual income of the people in a certain town has the following frequency distribution table.

Annual income ($1000)	No. of people (in 1000)
5	7
6	6
7	5
8	4
9	3
10	2
11	1

5. Example of a miscellaneous distribution (not belonging to Examples 1-4):

The opinions of the students in a certain school were investigated on the issue "Boys in the school are allowed to have long hair." The following is the result:

Answer	No. of students
SD (Strongly disagree)	100
D (Disagree)	450
U (Undecided)	300
A (Agree)	250
SA (Strongly agree)	100

6. Remarks:

6.1 The distributions from Example 1, 2, and 3 are "symmetrical" because we can draw a vertical line dividing a histogram into two identical halves.

6.2 The distributions from Example 4 and 5 are "asymmetrical" because we cannot find a vertical line which divides a histogram into two identical halves.

6.3 In practice we will not get exact shapes of distributions as illustrated by Examples 1-4, but we may approximate a shape of the frequency distribution referring to those names.

LESSON 4 EXPERIMENTAL DISTRIBUTION

PURPOSE: To introduce the basic concepts of:

1. Population and sample
2. Sampling
3. Using a sample distribution to predict a population distribution.

EXPERIMENT: Drawing a sample of size 50 from Box "B" Box "N" Box "T".

Box "B"

x	f
100	1
101	2
102	5
103	9
104	19
105	28
106	19
107	9
108	5
109	2
110	1
	N=100

Box "N"

x	f
1	1
2	2
3	3
4	4
5	7
6	9
7	12
8	15
9	18
10	19
11	20
12	19
13	18
14	15
15	12
16	9
17	7
18	4
19	3
20	2
21	1
	N=200

Box "T"

x	f
1	20
2	19
3	18
4	17
5	16
6	15
7	14
8	13
9	12
10	11
11	10
12	9
13	8
14	7
15	6
16	5
17	4
18	3
19	2
20	1
	N=210

MATERIAL:

1. A paper to record a sample.
2. A paper to construct a frequency distribution table and a histogram.

PROCEDURE:

1. Form a group of two.
2. Go to the location of one of Box "B" or Box "N" or Box "T" as the teacher's assignment.
3. One student in a group shakes the box, then blindly picks a piece of paper from the box and reads the

number to another student to record on a prepared paper.

4. Without returning any previous pieces of paper which have already been recorded, repeat "step 3" until you get fifty numbers on the recorded sheet.
5. Return all pieces of recorded numbers to the box, and another group will follow.
6. Use the data from the recorded sheet to construct:
    - 6.1 A frequency distribution table
    - 6.2 A histogram
    - 6.3 A frequency polygon
7. A set of data in each of Box "B" or Box "N" or Box "T" is called a "population."
8. A set of data collected from any population is called "a sample." As the example the set of data in your recorded sheet is a sample of a population in Box "B" or Box "N" or Box "T". The sample size is 50.
9. A procedure used to collect a sample is called "sampling." Step 3 and 4 are our procedure to collect a sample. It is called "sampling without replacement."
10. Usually the distribution of a population is not known. But we could observe the distribution of a sample taken from that population to predict or estimate the population distribution. Statisticians generally make the assumption about a population distribution beforehand, for instance, the heights of all grade nine students is a kind of bell shaped distribution as the assumption. However, to make sure that the assumption is safe, a sample distribution will be observed and checked later.
11. Observe the frequency polygon in "Step 6."
    - 11.1 Does the frequency polygon indicate some shape of distribution?
    - 11.2 What kind of a population distribution would you predict by observing the frequency polygon of a sample? (Referring to the shapes of distribution in Lesson 3.)

11.3 A sample size is 50. A population size: Box "B" is 100, Box "N" is 200, Box "T" is 210. The distribution of which box seems to be best estimated by a sample of size 50? Why?

12. Remarks:

12.1 The teacher is expected to elaborate the concepts of "population," "sample," and "sampling without replacement."

12.2 The teacher may recommend students to construct a frequency table by using the class interval of width "2", because if the class interval of width "1" does not clearly indicate the important feature of the distribution.

RECORDING PAPER

Score (x)			Tally	Frequency (f)
Box B	Box T	Box N		
100	1	1		
101	2	2		
102	3	3		
103	4	4		
104	5	5		
105	6	6		
106	7	7		
107	8	8		
108	9	9		
109	10	10		
110	11	11		
	12	12		
	13	13		
	14	14		
	15	15		
	16	16		
	17	17		
	18	18		
	19	19		
	20	20		
		21		

LESSON 5 COLLECTING PERSONAL DATA

PURPOSE: 1. To introduce some real situations in collecting data.

2. To motivate students by using their personal data to illustrate statistical concepts.

EXPERIMENT: -

MATERIAL: Eleven pieces of paper for collecting personal data.

1. Height (in nearest inch)
2. Weight (in nearest pound)
3. Age (in nearest month)
4. Approximated hours watching TV per week
5. Supper time (nearest 15 minutes)
6. Bed-time (nearest 15 minutes)
7. Getting up time (nearest 15 minutes)
8. Number of children in a family
9. Money allowance per week (in dollars)
10. Social studies marks (last term)
11. Shoe size.

PROCEDURE:
1. Every student gives his or her own information on a collected sheet of paper.
2. As a student's own choice or the teacher's assignment
   - 2.1 Select two sets of data to construct frequency tables
   - 2.2 Construct the histograms.
3. Describe approximately the shapes of distributions in "step 2."
4. We can view each set of data as a sample from a population. Will you agree if we view the procedure of collecting these samples as "sampling without replacement"? Why?
5. If we define a set of heights as a sample collecting from a population of the heights of all grade nine students in this school.

   Similarly how would you define the two sets of data you selected in "step 2"?

LESSON 6 CENTRAL TENDENCY

PURPOSE: To introduce the basic concept of the measure of central tendency.

1. Mean
2. Median
3. Mode

EXPERIMENT: -

MATERIAL: -

PROCEDURE: With any given data after a table of a frequency distribution has been constructed, generally a measure of a central tendency or a central position is often calculated. A measure of a central tendency will give us a concise description of a typical score of a set of data as a whole, or the value where the majority of data seems to cluster.

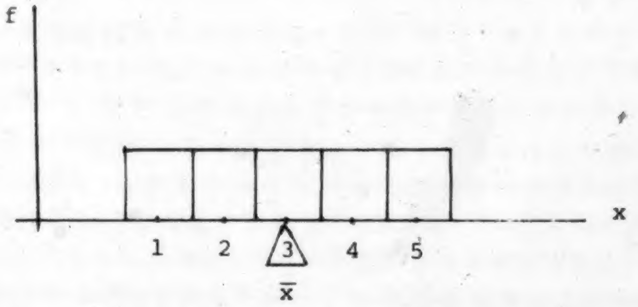
Mean

1. The first important measure of a central tendency is called "mean." We will use the symbol $\overline{x}$ (read "x bar") for "mean." The concept of "mean" is generally familiar, because it is the same as "average."
2. Example: A set of score: 1, 2, 3, 4, 5

   We can find the average = $\frac{(1+2+3+4+5)}{5} = \frac{15}{5} = 3$

   We know that 15 is the sum of total scores, 5 is the total number of scores.

   Hence in general formula: Mean = $\frac{?}{?}$
3. Look at the histogram of scores: 1, 2, 3, 4, 5 and the mean.

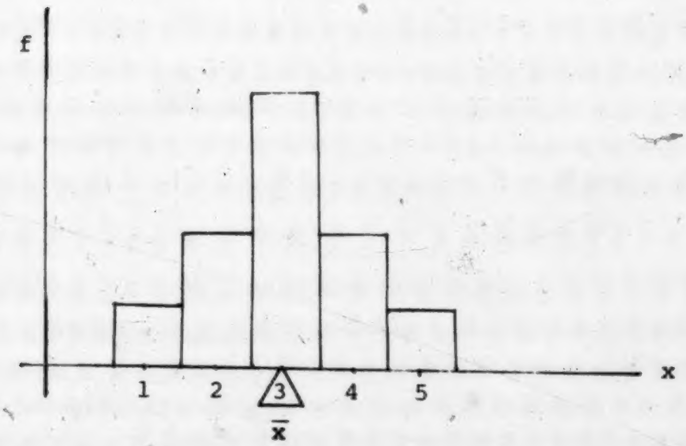


The mean is right at the central location of the distribution.

4. Example: Compute $\overline{x}$, draw the histogram and show the location of $\overline{x}$ in the histogram.

x	f	fx
5	1	5
4	2	8
3	4	12
2	2	4
1	1	1
	$\Sigma f=10$	$\Sigma fx=30$

$$x = \frac{\text{sum of scores}}{\text{total no. of scores}} = \frac{30}{10} = 3$$



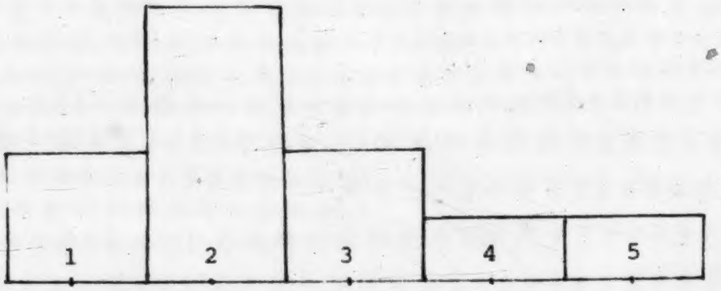
5. Observe both examples 2 and 4 have symmetrical distributions and both have the means right at the center position of the histograms. The location of the mean also represents the point of a fulcrum on which the histogram will stay level or balanced. In other

words "the mean" is also a center of gravity in a frequency distribution. (It is expected that the teacher will have to elaborate on the concept of center of gravity.)

6. Example: Complete the blanks then place a fulcrum (Δ) at the balance point of the histogram.

x	f	fx
5	1	..
4	1	..
3	2	..
2	4	..
1	2	..
	Σf=..	Σfx=..

$$\overline{x} = \frac{\ldots\ldots}{\ldots\ldots} = \ldots.$$



7. The distribution of Example 6 is asymetrical. Observe the histogram, the highest peak is towards the left end which means that the greatest number of scores is in the same direction.

   7.1 Observe the location of the fulcrum or the mean. Which direction is it from the center of the peak (2)?

   7.2 If we have another histogram whose highest peak was towards the right end, which direction is the mean from the peak?

   7.3 If the histogram has a long "tail" on one side on which side of the highest peak is the mean?

Median

8. The second measure of a central tendency is called "median." We will use the abbreviation "Mdn." The median is a middle score or a middle value when the data have already been arranged by magnitude.

9. Example: A set of scores: 2, 1, 4, 3, 5

Arrange in order: 1, 2, 3, 4, 5

The median is 3 because it is the middle score.

Obviously $\overline{x}$ is also 3.

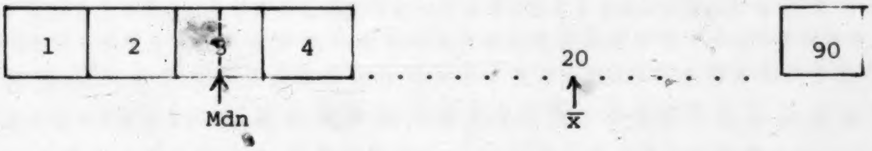
10. Example: A set of scores: 2, 1, 4, 3, 90

Arrange in order: 1, 2, 3, 4, 90

The median is 3.

But the mean or $\overline{x}$ is $(1+2+3+4+90)/5 = 20$.

11. Look at the histogram of Example 10 and the locations of median and mean.



11.1 In Example 9 both mean and median really represent the centrally located score. But in Example 10, as shown by the above histogram, the mean does not represent the centrally located score. It moves way out of the majority toward the extreme score, 90.

11.2 Observe the vertical line, which passes through the median, will divide the area of the histogram into two equal left and right parts.

Therefore, we can define a median as a point on which a vertical line can be drawn to divide a histogram into two halves.

12. Find the medians of the following distributions:

12.1 Example: 2, 4, 5, 1, 70

The scores in order: ....

Mdn = ...

12.2 Example: 4, 2, 6, 9

Rewrite the scores in order: 2, 4, 6, 9

The middle point of the distribution is between 4 and 6, which is $(4+6)/2 = 5$.

Mdn = 5.

12.3 Example: 13, 10, 20, 31

The arranging scores: ....

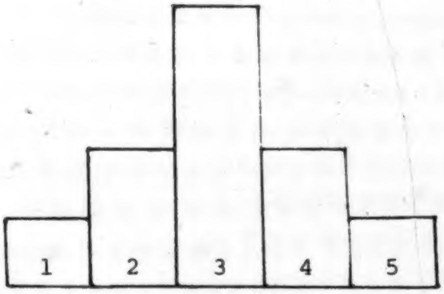
Mdn = ...

13. Referring back to Example 12:

13.1 What is different in finding medians from Example 12.1 and 12.2?

13.2 Construct a histogram for Example 12.2, mark the location of the median and observe if it divides the histogram into two halves.

14. Look at the following histogram and answer the questions:



14.1 The median is ....

14.2 The mean is ....

14.3 The distribution is symmetrical. Does this type of distribution indicate something about the mean and median?

Mode

15. Mode is another measure of a central tendency of a distribution. It is defined as a score or value which occurs with the most frequency, provided there exists one. In other words "Mode" is a score which has the most repetitions in the set of data.

16. Observe the illustrations:

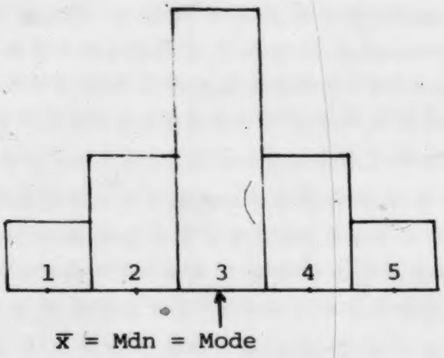
16.1 Example: The scores: 2, 3, 4, 4, 4, 5, 5

Mode is 4 (Why?).

16.2 Example: The scores: 1, 2, 3, 4, 5

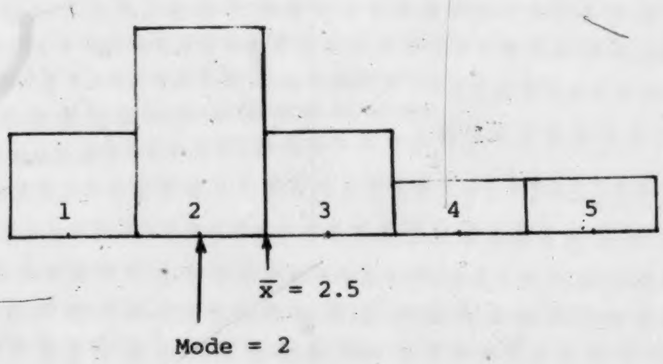
There is no mode (Why?).

16.3 Example: Find the mode of the following histogram (from Example 4):



The mode is 3 (Why?).

16.4 Example (from Example 6): Find the mode of the following histogram:



17. Observe:

17.1 The histogram in Example 16.3 is a bell-shaped distribution. All three measures of central tendency indicate the same value, right at the center of the highest peak.

17.2 The histogram in Example 16.4 is an asymmetrical distribution. The mode is right at the center of the highest peak while the mean and median are not.

18. Mode is useful when we would like to know the most characteristics value of a group.

18.1 Mode is more useful to measure a central tendency of a set of data such as that of shoe sizes sold in a department store.

18.2 Mode also assures that its value occurs as part of the data as in the example of shoe sizes. If mean or median are used, the value may not get a shoe size that really exists.

LESSON 7 ACTIVITY ON MEASURES OF CENTRAL TENDENCY

PURPOSE: To have the students use and practice the concepts they have learned on Mean, Median, and Mode.
(30 working; 10 minutes discussion.)

EXPERIMENT: -

MATERIAL:
1. A frequency distribution table and a histogram of a sample drawn from one of the three populations in Lesson 4.
2. Personal data collected in Lesson 5.

PROCEDURE: Do both 1 and 2 or as the teacher assigns.

1. Part A: From a frequency distribution table and a histogram of a sample drawn in Lesson 4:
   - 1.1 Compute the mean.
   - 1.2 Estimate the median.
   - 1.3 Estimate the mode.
   - 1.4 Mark the location of the mean, median, and mode in the histogram.
2. Part B: From two sets of data you have selected out of 11 sets of personal data:
   - 2.1 Compute the mean.
   - 2.2 Compute the median (to estimate).
   - 2.3 Compute the mode.
   - 2.4 What would be the best measure of central tendency for the two sets of personal data: shoe size, number of children in a family, in order to guarantee the existence of the value of the measure?
3. Remarks: Teacher discusses Part A and B and tries to focus on:
   - 3.1 Are there particular sets of data that are best measured by mode?
   - 3.2 Are there some sets of data that are best measured by median?
   - 3.3 Are there some sets of data that are equally best measured by $\overline{x}$, mdn. and mode?

LESSON 8 VARIABILITY

PURPOSE: To introduce the basic concept of the measure of variability.

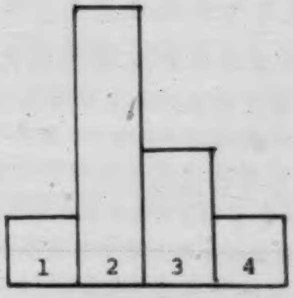
1. Range
2. Variance and standard deviation.

EXPERIMENT: -

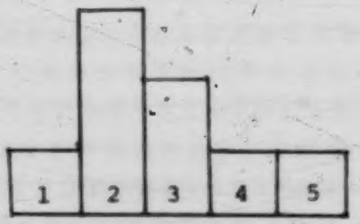
MATERIAL: -

PROCEDURE: Study and follow the following illustrations:

1. So far we have already studied the description of a set of data by:
   1.1 Frequency distribution
   1.2 Central tendency.
   There is another way of describing a set of data in order to have some index of how much spread or variability exists in the distribution of the given set of data.
2. Look at the following pairs of distribution, and judge (by your eyes) which one seems to be more variable than the other.
   2.1 Example



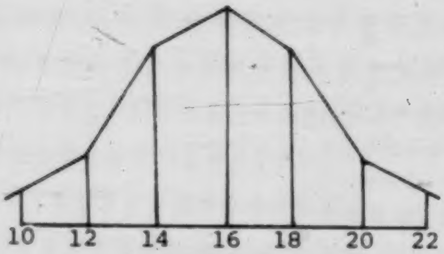
Histogram A



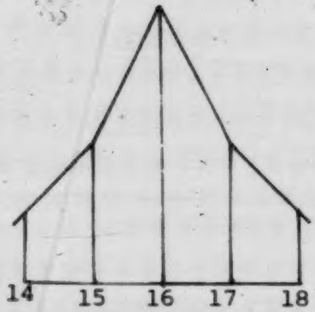
Histogram B

.... is more variable than ....

2.2 Example:



Frequency Polygon A



Frequency Polygon B

.... represents the most variable distribution.

2.3 Example: The heights of two groups of students (in centimeters)

Group A:- 145, 150, 160, 170, 175

Group B: 150, 160, 160, 160, 170

What is $\overline{X}$ for Group A and Group B?

.... and ....

The most variable group is ....

3. There are several ways to measure the spread or variability of a set of data. We will mention only two of them: Range and Standard deviation.

4. Range is the crude measure of the variability of data. It is simply the width or difference between the largest and smallest scores in a set of data.

Range = The largest score minus ....

4.1 Example: Find the ranges of the following scores:

a. 1, 3, 2, 5, 7

Range = 7-1 = 6

b. 1, 0, 3, 2, 5

Range = ...

c. -2, -1, 0, 1, 2

Range = ...

4.2 Only two scores in a set of data are used to determine the "range." These two scores are .... and ....

This is why the range is considered as the crude measure of the variability.

5. Standard deviation

   Standard deviation is the most efficient and useful measure of the variability. Every score in a set of data is used for calculating the measure. It indicates how much the data spreads out from the mean.

   We use SD or S to represent the standard deviation.

6. The following examples will illustrate how to calculate the standard deviation:

   6.1 Example: Find the standard deviation of 1, 2, 3, 4, 5.

   First we find $\overline{x} = (1+2+3+4+5)/5 = 3$ (N = 5)

Scores x	Deviation $(x - \overline{x})$	Squared Deviation $(x - \overline{x})^2$
1	$(1-3)=-2$	$(-2)^2= 4$
2	$(2-3)=-1$	$(-1)^2= 1$
3	$(3-3)= 0$	$0^2= 0$
4	$(4-3)= 1$	$1^2= 1$
5	$(5-3)= 2$	$2^2= 4$
	$\Sigma(x-\overline{x})= 0$	$\Sigma(x-\overline{x})^2= 4+1+0+1+4 = 10$

$$SD^2 = \frac{\Sigma(x-\overline{x})^2}{N} = \frac{10}{5} = 2$$

$$SD = \sqrt{2} = 1.414$$

   6.2 From the illustration in Example 6.1, we can write down the steps for calculating the standard deviation as follows:

   a. Find $\overline{x}$ or mean of the data.

   b. Find $(x-\overline{x})$ or the difference between each score and the mean.

      Observe the sum of all differences or $\Sigma(x-\overline{x})$, is it always zero?

   c. Find the square of each difference or $(x-\overline{x})^2$ in Step B.

   d. Find the sum of all scores in Step C. Does the result in Step C have a chance to be a negative number? Why?

   e. Find SD^2 which is called "variance." The variance is the average of the scores in Step C. It is the same as the result in Step D divided by the total number of scores. Is it the same $SD^2 = \frac{\Sigma(x-\overline{x})^2}{N}$?

f. Find SD or the standard deviation from Step E. What is the relation between the standard deviation and the variance of the same set of scores?

6.3 Example: Find the standard deviation from the table of frequency distribution.

x	f	fx	d $(x-\overline{x})$	d^2	fd^2
5	1	5	2	4	1x4=4
4	2	8	1	1	2x1=2
3	4	12	0	0	4x0=0
2	2	4	-1	1	2x1=2
1	1	1	-2	4	1x4=4
	$\Sigma f=10$	$\Sigma fx=30$			$\Sigma fd^2=4+2+0+2+4=12$

$$\overline{x} = \frac{\Sigma fx}{\Sigma f} = \frac{30}{10} = 3$$

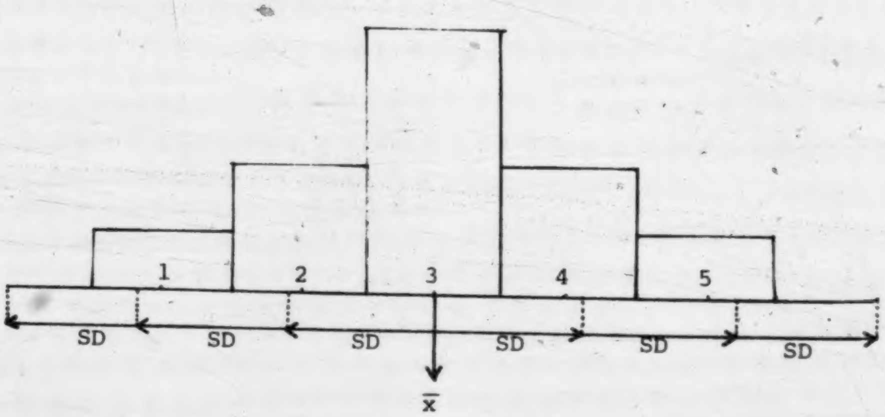
Variance or $SD^2 = \frac{\Sigma fd^2}{N} = \frac{12}{10} = 1.2$

$SD = \sqrt{\text{Variance}} = \sqrt{1.2} = 1.1$ (Approx.)

The steps for calculating for Standard deviation are the same as illustrated in Example 6.1, but in Example 6.3 some scores have more frequencies than one.

6.4 Look at the histogram of the frequency distribution in Example 6.3, and observe how the standard deviation is used to measure the spread. It starts at the location of the mean or $\overline{x}$, then measure can be made to the left or right ends of the histogram.

Standard deviation has the same unit of measure as the original data. In Example 6.3, suppose the scores are measured in "inches," then the standard deviation = 1.1 inch.



*6.5 There are three groups of students whose scores in mathematics test as follows:

Group A		Group B		Group C	
x	f	x	f	x	f
80	1	80	1	80	1
70	1	60	1	70	1
65	1	58	1	55	6
60	1	56	1	40	1
55	2	55	2	30	1
50	1	54	1		
45	1	52	1		
40	1	50	1		
30	1	30	1		

a. Find the means:

$\overline{x}$ of Group A = ...

$\overline{x}$ of Group B = ...

$\overline{x}$ of Group C = ...

Could you tell the differences among the groups of students by just comparing their means? Why?

b. Find the ranges:

The range of Group A = ...

The range of Group B = ...

The range of Group C = ...

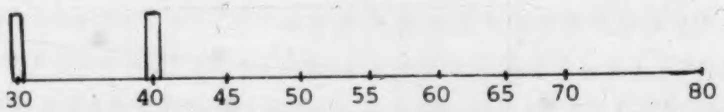
Does the result of obtaining the ranges indicate the differences of the variability among the three groups? Why?

---

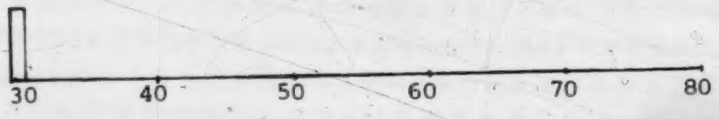
*This is to be done by the students with guidance.

c. Draw the histograms:

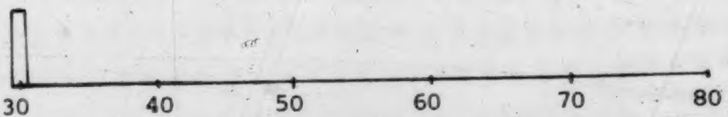
Complete the histograms:



Group A



Group B



Group C

By looking at these histograms:

What group seems to represent the most variable group? ....

What group seems to represent the least variable group? ....

d. Find the standard deviations of the three groups.

What group has the biggest standard deviation?

What group has the smallest standard deviation?

Compare the above answers with the answers of Section C.

Does the biggest standard deviation represent the most variable group?

Does the smallest standard deviation represent the least variable group?

Do you think the standard deviation is a more efficient measure than the range?

7. Generally we can not judge the variability of two sets of data by our eyes, especially when both sets are very similar and consist of many data. The standard deviations (or variances) of the two sets will tell the different variability.

LESSON 9 ACTIVITY ON MEASURES OF VARIABILITY

PURPOSE: To have the students use and practice the concepts they have learned on a range and a standard deviation.

EXPERIMENT: -

MATERIAL: 1. A frequency distribution table from Lesson 4, and the result of finding its mean from Lesson 7.

2. Personal data collected in Lesson 5, and the means which have already been computed in Lesson 7.

PROCEDURE: Do both 1 and 2 or as the teacher assigns.

1. Part A: From a frequency distribution table of a sample drawn in Lesson 4.
   1.1 Find the range
   1.2 Find the variance
   1.3 Find the standard deviation.
2. Part B: From two sets of data you selected out of 11 sets of personal data and you have already computed the means:
   2.1 Find the ranges
   2.2 Find the variances
   2.3 Find the standard deviations.
3. Remarks:
   3.1 Table of squares and square roots should be available.
   3.2 The teacher may have a small electronic calculator.
   3.3 The teacher may decide to omit some exercises that take too much time.

LESSON 10 MEAN AND STANDARD DEVIATION IN TRANSFORMED SCORES

PURPOSE: To introduce the basic concepts of how mean and standard deviation change in transformed scope.

1. If a set of scores is added by a constant.
2. If a set of scores is multiplied by a constant.
3. Standard score.

EXPERIMENT: -

MATERIAL: -

PROCEDURE:

Part A: If a set of scores is transformed to another set by adding or subtracting a constant.

1. Look at the following tables:

x	f
1	3
2	5
3	1
4	1
	N=10

Table 1

x'	f
6	3
7	5
8	1
9	1
	N=10

Table 2

x"	f
-5	3
-4	5
-3	1
-2	1
	N=10

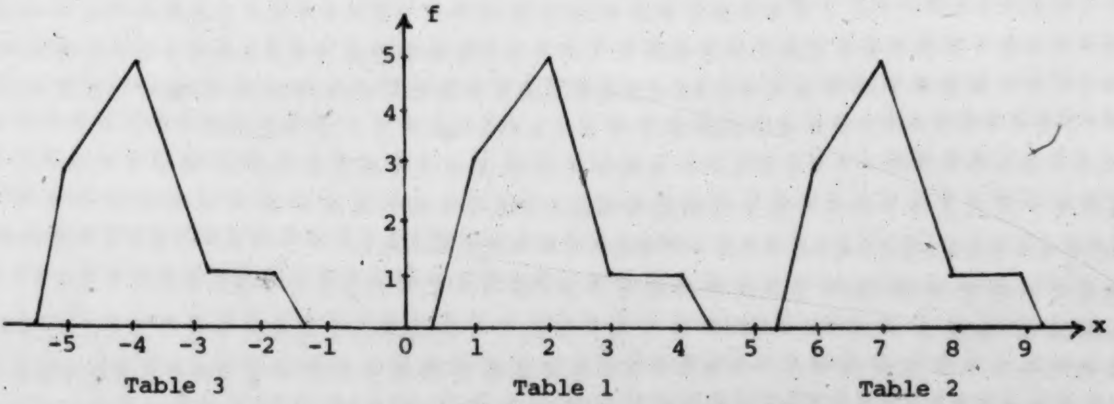
Table 3

2. You will observe that Table 2 and Table 3 are transformed from Table 1.

   2.1 Table 2 is transformed from Table 1 by adding 5.

   2.2 Table 3 is transformed from Table 1 by subtracting 6.

3. Look at frequency polygons of the three tables:



4. Observe the shapes of the three frequency polygons. All three look identical (Do they?). However, they have different positions.
5. Do you agree that the means of the three tables are different? Because the three frequency polygons have different locations on the number line.
6. Do you see that the standard deviations of the three tables are not different? Because they have identical shapes of frequency polygons.
7. Look at the computation for means and standard deviations of the three tables.

Table 1:

x	f	fx	d $(x-\bar{x})$	d^2	fd^2
1	3	3	-1	1	3
2	5	10	0	0	0
3	1	3	1	1	1
4	1	4	2	4	4
	N=10	$\Sigma fx=20$			$\Sigma fd^2=8$

$$\bar{x}_1 = \frac{\Sigma fx}{\Sigma f} = \frac{20}{10} = 2$$

$$s_1^2 = \frac{\Sigma fd^2}{\Sigma f} = \frac{8}{10} = .8$$

$$s_1 = \sqrt{.8} = .9 \text{ (Approx.)}$$

Table 2:

x'	f	fx'	d	d^2	fd^2
6	3	18	-1	1	3
7	5	35	0	0	0
8	1	8	1	1	1
9	1	9	2	4	4
	10	70			8

$$\bar{x}_2 = \frac{70}{10} = 7$$

$$s_2^2 = \frac{8}{10} = .8$$

$$s_2 = \sqrt{.8} = .9$$

Table 3:

x"	f	fx"	d	d^2	fd^2
-5	3	-15	-1	1	3
-4	5	-20	0	0	0
-3	1	-3	1	1	1
-2	1	-2	2	4	4
	10	-40			8

$$\bar{x} = \frac{-40}{10} = -4$$

$$S_3{}^2 = \frac{8}{10} = .8$$

$$S_3 = \sqrt{.8} = .9$$

8. The results of step 7 show that

   8.1 All the three means ($\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$) are different.
   The mean of Table 2 is equal to the mean of Table 1, plus 5 (Compare 2.1).
   The mean of Table 3 is equal to the mean of Table 1, minus 6 (Compare 2.2).

   8.2 All the three standard deviations (S_1, S_2, S_3) are the same.

9. Conclusion:

   If a new set of scores are received from an old set of scores by adding a given number then

   9.1 the mean of a new set is equal .... plus that given number

   9.2 the standard deviations of the two sets are ....

Part B: If a set of scores is transformed to another set by multiplying or dividing by a constant.

10. Look at the following tables:

x	f
1	3
2	5
3	1
4	1
	N=10

Table 1

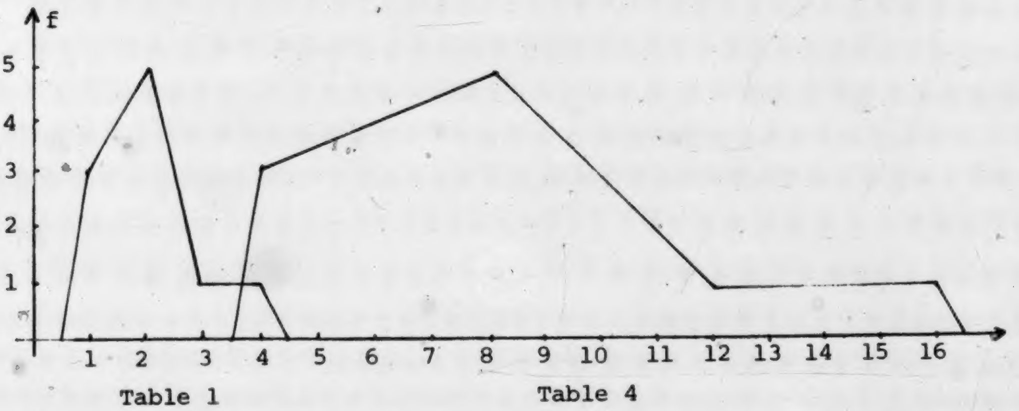
x	f
4	3
8	5
12	1
16	1
	N=10

Table 4

11. Observe that Table 4 is transformed from Table 1. Table 4 is transformed from Table 1 by multiplying every score by 4.

2

12. Look at the frequency polygons of Table 1 and 4.



13. Observe the two frequency polygons.
    - 13.1 The locations are different (Table 4 moves to the right of Table 1).
    - 13.2 The shapes are different (Table 4 is wider than Table 1).

14. As the result of the observation (13.1 and 13.2):
    - 14.1 Do you see that the means of Table 1 and 4 are different?
    - 14.2 Do you see that the standard deviations of Table 1 and 4 are different?

15. Look at the computation for means and standard deviations of Table 1 and 4.
    - 15.1 Table 1 has already been computed.

$\bar{x}_1 = 2 \qquad S_1^2 = .8 \qquad S_1 = .9$

- 15.2 For Table 4:

x"	f	fx"	d	fd	fd^2
4	3	12	-4	-12	48
8	5	40	0	0	0
12	1	12	4	4	16
16	1	16	8	8	64
	N=10	80			128

$$\bar{x}_4 = \frac{80}{10} = 8$$

$$S_4^2 = \frac{128}{10} = 12.8$$

$$S_4 = \sqrt{12.8} = 3.6$$

16. Observe the result of Step 15 and keep in mind that Table 4 = Table 1 multiplied by 4 (Step 10 and 11).

16.1 $\bar{x}_4$ (the mean of Table 4) = $\bar{x}_1$ (the mean of Table 1) multiplied by 4.

16.2 S_4 (the standard deviation of Table 4) = S_1 (the standard deviation of Table 1) multiplied by 4.

17. Conclusion: If a new set of scores are received with an old set of scores by multiplying with a given number, then

17.1 the mean of the new set is equal to the mean of the old set .... that given number.

17.2 the standard deviation of the new set = the standard deviation of the old set .... that given number.

Part C: Standard Scores

18. Look at Table 5 as the example:

x	f
5	3
10	5
15	1
20	1
	N=10

$\bar{x} = 10$

SD = 4

Table 5 is transformed from Table 1 by multiplying by 5. Therefore we can figure out the mean and standard deviation of Table 5 from the known mean and standard deviation of Table 1.

19. We can transform Table 5 to another table called "Table 6" by subtracting the mean of Table 5 or 10 from every score in Table 5 as the result.

x	f
-5	3
0	5
5	1
10	1
	N=10

$\bar{x} = 0$

SD = 4

Table 6

20. We can transform Table 6 to another table called "Table 7" by dividing every score in Table 6 with its standard deviation or 4 as the result.

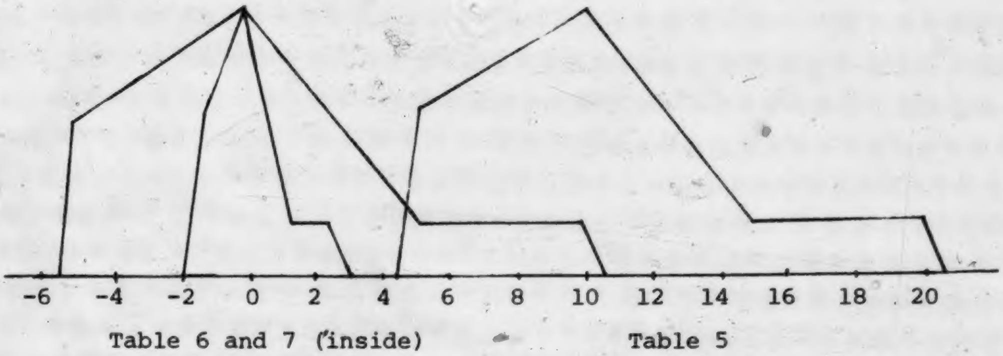
x	f
-1.25	3
0	5
1.25	1
2.5	1
	N=10

$\overline{x} = 0$

SD = 1

Table 7

21. Look at the frequency polygon of Table 5, 6, and 7.



21.1 Table 5 and Table 6 look exactly the same shape of the frequency polygons but the location of the mean has moved to zero.

21.2 Table 6 and Table 7 look different, but they have the same location of the mean which is zero. The difference in the shapes of the frequency polygons due to the difference of the standard deviations, SD of Table 6 is 4 and SD of Table 7 is 1.

22. The method we use in Step 19 and 20 is called to standardize scores or data into standard score. If we combine the two steps together we will get the formula:

$$\text{a standard score} = \frac{\text{a score - the mean}}{\text{standard deviation}} \text{ or } z = \frac{x - \overline{x}}{SD}$$

If we standardize Table 1, 2, 3 and 4 the result will be exactly as Table 7. Try some of them.

LESSON 11 ACTIVITY ON TRANSFORMED SCORES

PURPOSE: To practice using the concepts of transformed scores to various situations.

EXPERIMENT: -

MATERIAL: Personal data (from Lesson 5).

PROCEDURE: Do the following exercises as the teacher assigns:

1. What were the mean and standard deviation of the ages for all the students in this class five years ago? What will be the mean and standard deviation of the ages for the same group of students four years from now?
2. What would be the mean and standard deviation for the number of minutes in watching TV per week for all grade nine students in the class?
3. What would be the means and standard deviations for
   3.1 bed-time (in E.S.T.) for all students in this class?
   3.2 supper-time (in E.S.T.) for all students in this class?
   3.3 getting-up-time (in E.S.T.) for all students in this class?
4. If a set of money allowance per week for all students in this class is transformed to another set of scores with a zero mean
   4.1 What is the new score for the biggest money allowance?
   4.2 What is the new score for the least money allowance?
   4.3 What are the meanings of the new scores in 4.1 and 4.2?
5. What are the standard scores of the following:
   5.1 for your height from the heights of all the students in the class?
   5.2 for your weight from the weights of all the students in the class?
   5.3 Compare the results of 5.1 and 5.2 and make interpretation, for instance of 5.1 is 3, of 5.2 is -1, indicate that you are very tall but thin compared with all students in the class.

LESSON 12 NORMAL DISTRIBUTION

PURPOSE: To introduce the basic concepts of a normal distribution.

EXPERIMENT: -

MATERIAL: -

PROCEDURE: Study and follow the illustrations.

1. Example: Given the frequency distribution table of the heights (in centimeters) of 200 people

x (height)	f (number of people)	x	f
151	1	166	16
152	1	167	13
153	1	168	11
154	2	169	10
155	3	170	9
156	4	171	8
157	5	172	7
158	7	173	5
159	8	174	4
160	9	175	3
161	10	176	2
162	11	177	1
163	13	178	1
164	16	179	1
165	18		
			N=200

2. Look at the frequency polygon of Example 1.

   2.1 We observe that it is a bell shaped distribution.

   2.2 We know that a bell-shaped distribution has the mean, median, and mode with the same value.
   In this case the mean is ....

   2.3 We know that every set of data must have a standard deviation.
   In this case the standard deviation is approximately 5.

   2.4 We know that a vertical line passing through the mean and the highest peak will divide the area of the polygon into two identical halves.

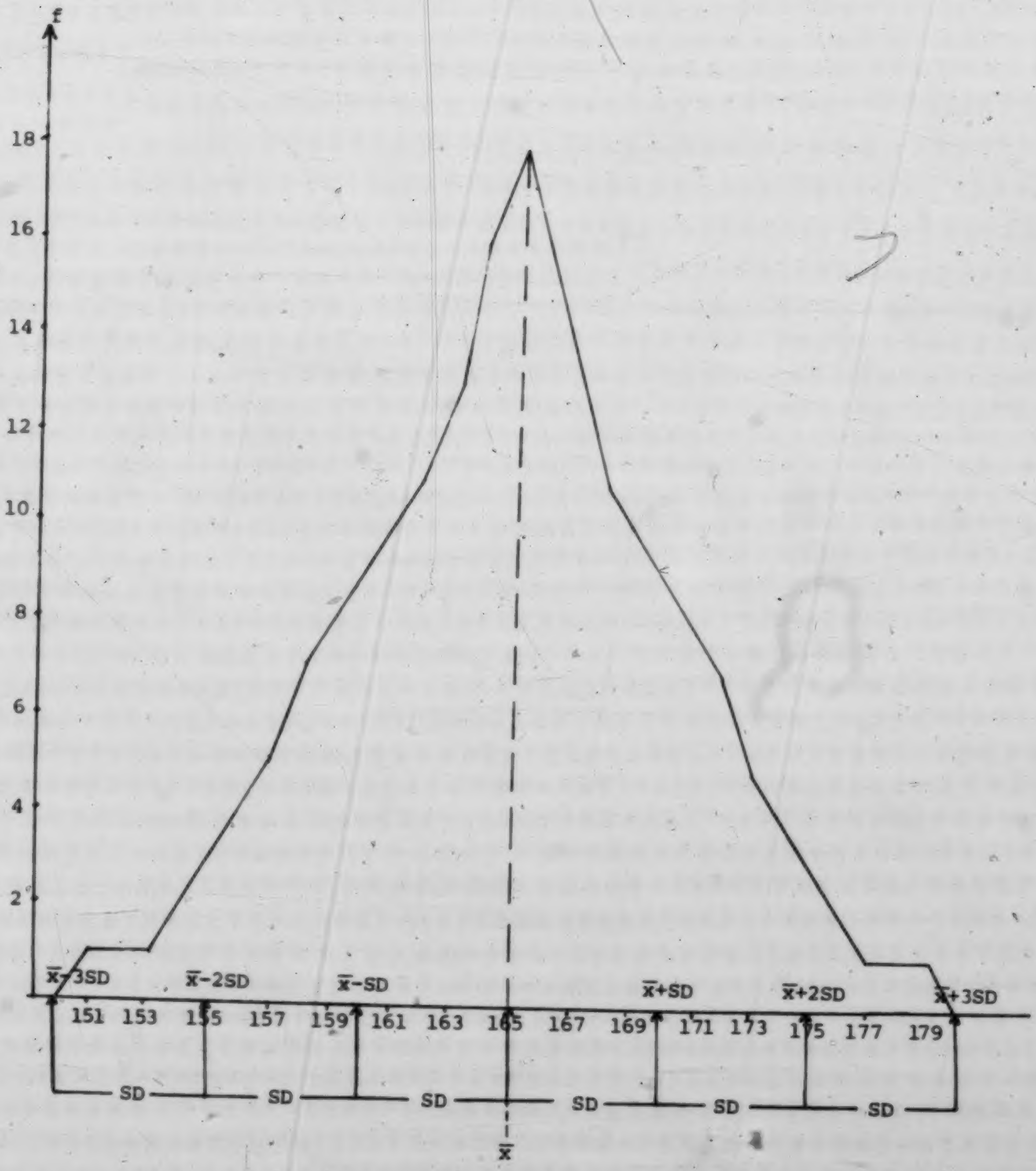


Figure 1A

2.5 Observe Figure 1A. There are three standard deviations to either left or right of the mean to cover almost all data in the distribution.

3. From Example 1, complete the following:

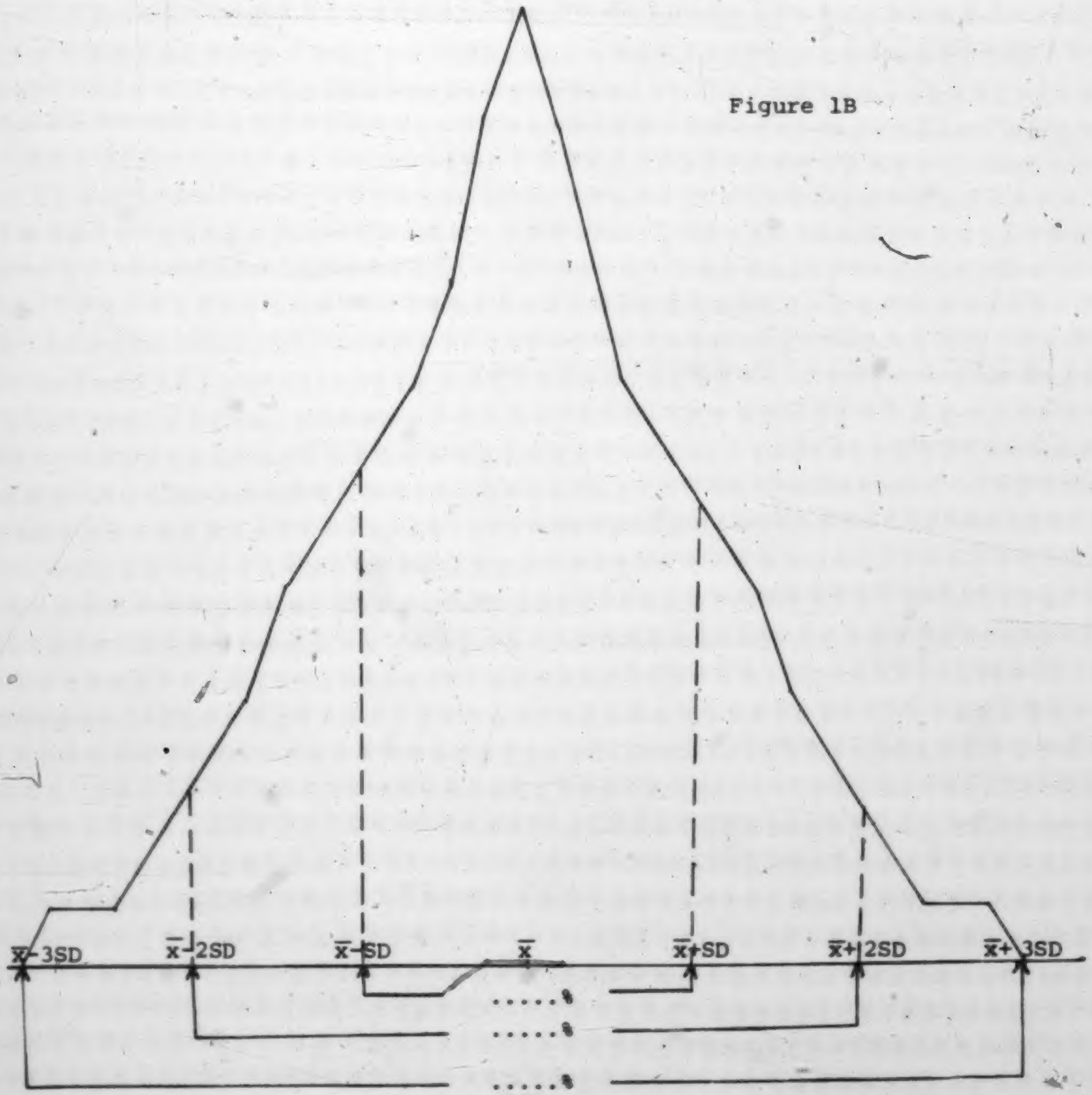


Figure 1B

3.1 SD = 5

$\bar{x}$ = ...

3.2 How many people from the total 200 are included in the intervals:

a. one standard deviation below and above the mean? ....

b. two standard deviations below and above the mean? ....

c. three standard deviations below and above the mean? .....

3.3 What are the percentages of the total included in the intervals from 3.2?

a. ....%

b. ....%

c. ....%

3.4 The above frequency distribution approximately describes what is so called "a normal distribution."

4. Normal distributions may not look like the common shape, because they may have different means and standard deviations.

4.1 From Example 1, the heights of a group of 200 people were measured in centimeters and the frequency distribution is normal. Now if the heights of the same group of 200 people were measured in inches then the frequency distribution is still normal. However, the means and standard deviations are different. Then both normal distributions do not look like the common shape.

4.2 Suppose the head breadths and nose breadths of a large group of people were measured in centimeters. Suppose the frequency distribution for the head breadths is normal and also for the nose breadths. The means and standard deviations for the head breadths are 14 cm and .5 cm, and for the nose breadths are 4 cm and .3 cm, These two distributions do not look the same because the distribution for the nose breadths has a narrower shape than for the head breadths.

4.3 We have learned how to transform a set of scores to another set of scores by a certain formula. We can transform any normal distribution to another normal distribution by using the formula $z = \frac{x-\bar{X}}{SD}$. For example 4.2: the head breadths will be transformed by using $z = \frac{x-14}{.5}$; the nose breadths will be transformed by using $z = \frac{x-4}{.3}$. The two distributions are both still normal and now have the common shape. Both means = 0, standard deviation = 1.

4.4 A normal distribution using a standard unit as described in 4.3 will always have the mean = 0, the standard deviation = 1. It may be called a "normal distribution using standardized scores."

5. Normal distribution is very important and most often used in statistics. Here are some of the uses:

5.1 Biological measurements: Natural phenomena such as heights, weights, and intelligence were found possessing normal distributions.

For example, the weights of a great number of first born babies will possess a normal distribution; the scores of all grade nine students in Alberta on some standardized test will possess a normal distribution; and the arm length of a big number of men will also possess a normal distribution.

5.2 Sample means: The distribution of sample means taken from a population is normal. This idea will be illustrated later.

LESSON 13 DISTRIBUTION OF THE SAMPLE MEANS

PURPOSE: To introduce the basic concepts of the distribution of the sample means taken from a normal population through experimenting.

EXPERIMENT: Drawing random sample of size 2 from Box N.

Box "N"

x	f
1	1
2	2
3	3
4	4
5	7
6	9
7	12
8	15
9	18
10	19
11	20
12	19
13	18
14	15
15	12
16	9
17	7
18	4
19	3
20	2
21	1
	N=200

MATERIAL:
1. Box "N" containing square railroad papers printed assigned numbers.
2. Recording Sheet 1 and 2.
3. Graph paper.

PROCEDURE:

1. Form a group of two.
2. Go to the location of Box "N" as the teacher's assignment.
3. Draw 10 random samples of size 2 "with replacement" and record the result on Recording Sheet 1. Follow the following procedure:
   3.1 One student shakes the box, without peeping picks one piece of paper out of the box, then reads the number on that

piece of paper to another student and records it on Recording Sheet 1.

3.2 Return the first piece of paper to the box, then repeat Step 3.1 again.

3.3 You will now have two numbers, for instance the first pick is 10 and the second is 12, which is called a sample of size 2, or a size -2- sample.

3.4 Suppose the sample is (10, 12). You tally slash or "/" under column number 10 and 12 of Sheet 1.

3.5 You need 10 samples of size 2.

4. Complete Recording Sheet 2 by recording first ten means ($\overline{x}$) from your own Recording Sheet, and another 90 means from any of your nine different classmates.

5. Draw the histogram for the 100 means of samples of size 2 which were recorded on Recording Sheet 2.

6. Observe the histogram.
Does the histogram for the 100 means of size -2- random samples tend to be a bell shaped distribution?

7. Draw the smooth bell shaped curve to approximate the histogram.

8. Check roughly if the curve approximates the normal distribution.

8.1 Approximate the location of the mean of the curve.

8.2 Divide either the left or right of the mean into three equal parts.

8.3 Check if between the intervals

a. one standard deviation below and above the mean approximately include 68 cases

b. two standard deviations below and above the mean approximately include 95 cases

c. three standard deviations below and above the mean approximately include 100 cases.

9. Consider the procedure—we collect a size -2- sample from Box N, we use "sampling with replacement" procedure. Therefore, we never run out of data in Box N. This means we can have as many size -2- samples as we want.

9.1 We only use 100 random samples to construct the histogram

which already tends to describe a normal distribution.
(Does it? as the result of 8.)

9.2 Imagine if we keep increasing the number of samples as many as we want, would the histogram be very close to, and finally a normal distribution?

9.3 Imagine if we change a sample size instead of 2, would the result be a normal distribution as well?

10. Conclusion:

The population (Box N in this case) has a normal distribution, then the distribution of the sample means, based on a random sample of size n, is also ....

RECORDING SHEET 1

SAMPLE OF SIZE

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	$\bar{x}$
Example 1										/	/											10.5
Example 2						/	/		/		/	/										9
Sample 1																						
Sample 2																						
Sample 3																						
Sample 4																						
Sample 5																						
Sample 6																						
Sample 7																						
Sample 8																						
Sample 9																						
Sample 10																						

RECORDING SHEET 2

LESSON 13, 14

$\bar{x}$	Tally	f	$\bar{x}$	Tally	f	$\bar{x}$	Tally	f	$\bar{x}$	Tally	f
1			6			11			16		
1.2			6.2			11.2			16.2		
1.4			6.4			11.4			16.4		
1.5			6.5			11.5			16.5		
1.6			6.6			11.6			16.6		
1.8			6.8			11.8			16.8		
2			7			12			17		
2.2			7.2			12.2			17.2		
2.4			7.4			12.4			17.4		
2.5			7.5			12.5			17.5		
2.6			7.6			12.6			17.6		
2.8			7.8			12.8			17.8		
3			8			13			18		
3.2			8.2			13.2			18.2		
3.4			8.4			13.4			18.4		
3.5			8.5			13.5			18.5		
3.6			8.6			13.6			18.6		
3.8			8.8			13.8			18.8		
4			9			14			19		
4.2			9.2			14.2			19.2		
4.4			9.4			14.4			19.4		
4.5			9.5			14.5			19.5		
4.6			9.6			14.6			19.6		
4.8			9.8			14.8			19.8		
5			10			15			20		
5.2			10.2			15.2			20.2		
5.4			10.4			15.4			20.4		
5.5			10.5			15.5			20.5		
5.6			10.6			15.6			20.6		
5.8			10.8			15.8			20.8		
									21		

LESSON 14 ESTIMATION

PURPOSE: To introduce the basic concept of using a sample mean to estimate a population mean.

EXPERIMENT: Drawing random sample of size 5 from Box "N".

MATERIAL:
1. Box "N"
2. Recording Sheet 1 and 2
3. Graph paper to construct a histogram
4. A histogram of size 2 from Lesson 13.

PROCEDURE:

1. Draw 10 random sample of size 5 "with replacement" and record the result on Recording Sheet 1.
2. Complete Recording Sheet 2 by recording from 10 different Recording Sheet 1.
3. Draw the histogram for the 100 means of the samples of size 5.
4. As you have learned from Lesson 13 the histogram in Step 3 also tends to be normal. (Is it?)
5. Observe the shapes of both histograms for size 2 and 5 samples.
   - 5.1 Do both histograms seem to have the highest peaks over the population mean? (In this case is "11", a mean of Box "N"?)
   - 5.2 Does the bigger sample size tend to create the narrower shape of normal distribution?
   - 5.3 Does the narrower shape of the normal distribution tend to have more data close to a population mean?
   - 5.4 Complete the following blanks:
     - a. Between 9 to 13 (inclusive) there are .... cases for sampling distribution size 2.
     - b. Between 9 to 13 (inclusive) there are .... cases for sampling distribution of size 5.

     We try to find the number of data around a population mean (11) by using the same interval between 9 to 13 but with a different sample size distribution.

     Compare the results of "a" and "b" if the distribution of a bigger size sample has more cases in the same interval around a population mean.

6. Conclusion: From Step 5
   - 6.1 We use a sample mean to estimate a population mean, because generally a population mean is not known.
   - 6.2 A distribution of sample means will cluster around a population mean. If we find a mean of a sampling distribution we will get approximately a population mean. (5.1)
   - 6.3 The bigger a random sample size is, the better a sample mean is the estimate of a population mean. (5.2-5.4)
7. Remarks: The teacher may elaborate the idea from 5.4 as following:
   - 7.1 Estimate the intervals from the histograms:
     - a. What is the interval of $\overline{x} \pm 2SD$ for the distribution of size 2 sample means?
     - b. What is the interval of $\overline{x} \pm 2SD$ for the distribution of size 5 sample means?
   - 7.2 Since the distribution of sample means is normal, both the intervals in 7.1a and 7.1b include approximately 95% of the total.
   - 7.3 Since the mean of the distribution of sample means is close to the population mean, both the intervals of 7.1a and 7.1b would include the population mean. (Observe from the histogram.)
   - 7.4 Compare the width of the intervals in 7.1a and 7.1b. The width of 7.1b should be shorter, indicating that 95% of size 5 sample means are closer to the population mean than 95% of size 2 sample means. Hence a size 5 sample mean is a better estimate of a population mean than a size 2 sample mean.

LESSON 15 APPLICATION

PURPOSE: To introduce the basic concepts of applying some statistical knowledge that the students have learned to various situations.

EXPERIMENT: Drawing a sample from Box "B".

MATERIAL:
1. Box "B" (from Lesson 4)
2. Personal data (from Lesson 5).

PROCEDURE:

1. Generally there are four steps in statistical procedure:

   Step 1: Collection of data.
   We have learned two techniques of collecting data: sampling with replacement and sampling without replacement.

   Step 2: Presentation of data.
   We have learned how to organize data into a frequency distribution table, constructing a histogram and a frequency polygon.

   Step 3: Analysis of data.
   We have learned how to measure a central tendency and a variability. We also have learned how to use a distribution in a sample to check the assumption of a population distribution.

   Step 4: Interpretation of data.
   We have learned how to use a sample mean to estimate a population mean.

   You will apply the above knowledge to some situations in Part A and B.

2. Part A: For Box "B"
   - 2.1 Box "B" is a population which possesses a normal distribution.
   - 2.2 Collect a sample by using sampling with replacement. (Remember you have to judge a sample size by yourself.)
   - 2.3 Construct a frequency distribution table for a sample.

2.4 Compute a sample mean.

You may also check a population distribution by investigating the shape of frequency distribution table for a sample.

2.5 Estimate a population mean. (What would be the estimate of the mean for Box "B"?)

2.6 Check your answer with the teacher to determine how close you are.

3. Part B: For Personal Data

3.1 Suppose that a population for every set of data is normal.

3.2 Estimate the following:

a. The (average) height of grade nine students in this school.

b. The (average) weight of grade nine students in this school.

c. The age of grade nine students in this school.

d. The hours of watching TV for grade nine students in this school.

e. Supper-time for grade nine students in this school.

f. Bed-time for grade nine students in this school.

g. Getting-up-time for grade nine students in this school.

h. Number of children in a family of grade nine students in this school.

i. Money allowance per week per student for grade nine students in this school.

3.3 You can not check the answers of Step 8. Why? It is the nature of the statistical estimation.

APPENDIX B

EXAMPLES FROM THE STUDENTS' COPY: LESSONS ON BASIC INFERENTIAL STATISTICS

LESSON 1 FREQUENCY DISTRIBUTION

1. The heights of 30 grade nine students are measured in centimeters with the following results:

162	166	163	164	167	165	168	166	164	165
163	169	166	164	167	165	168	166	167	166
166	170	169	164	167	165	168	166	168	167

2. Complete the following table by using the above data.

Student Height x	Boundaries	Tally	Number of Students Within These Boundaries or Frequency (f)
170			
169			
168			
167			
166			
165			
164			
163			
162			

Total frequency = ...

3. Complete the table of the frequency distribution.

x	f
170	
169	
168	
167	
166	
165	
164	
163	
162	
	Σf = ...

4. Complete the following tables:

4.1

Student Height (x)	Boundaries	Frequency (f)
169 or 170		
167 or 168		
165 or 166		
163 or 164		
161 or 162	160.5 - 162.5	1
		Σf = ...

4.2

Student Height (x)	Boundaries	Frequency (f)
168 - 170		7
165 - 167	164.5 - 167.5	
		Σf = ...

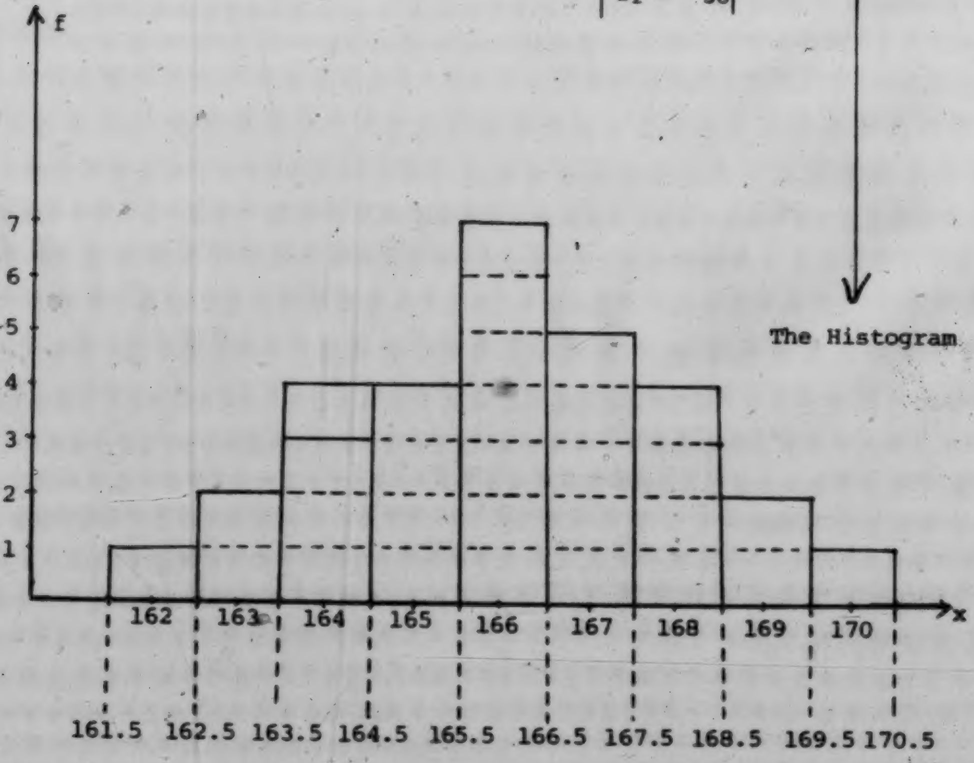
LESSON 2 GRAPH OF THE FREQUENCY DISTRIBUTION

1. A Histogram

1.1

x	Boundaries	f
170	169.5 - 170.5	1
169	168.5 - 169.5	2
168	167.5 - 168.5	4
167	166.5 - 167.5	5
166	165.5 - 166.5	7
165	164.5 - 165.5	4
164	163.5 - 164.5	4
163	162.5 - 163.5	2
162	161.5 - 162.5	1
		$\Sigma f =$ 30

The Table of the Frequency Distribution

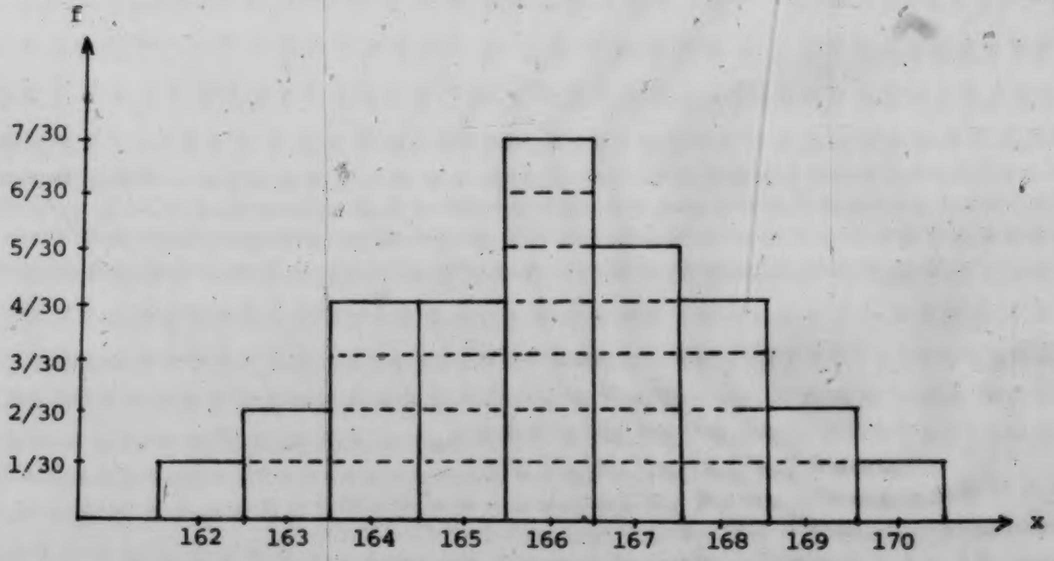


The Histogram

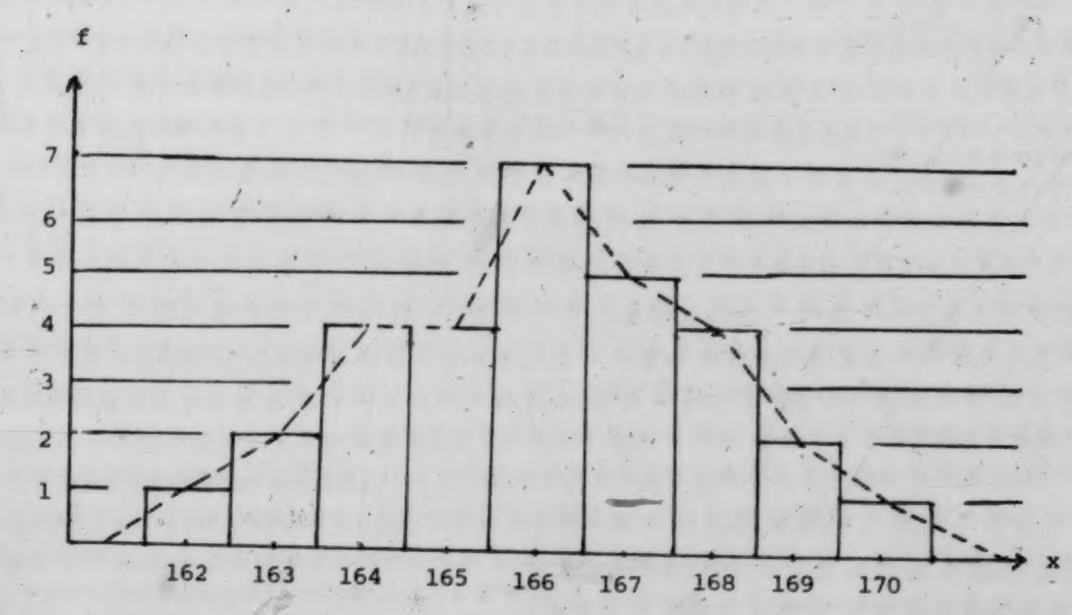
1.2 A histogram using the relative frequences (f/N)

x	f	f/N
170	1	1/30
169	2	2/30
168	4	4/30
167	5	5/30
166	7	7/30
165	4	4/30
164	4	4/30
163	2	2/30
162	1	1/30

$N=\Sigma f=30$ $\Sigma f/N=1$



2. A frequency polygon



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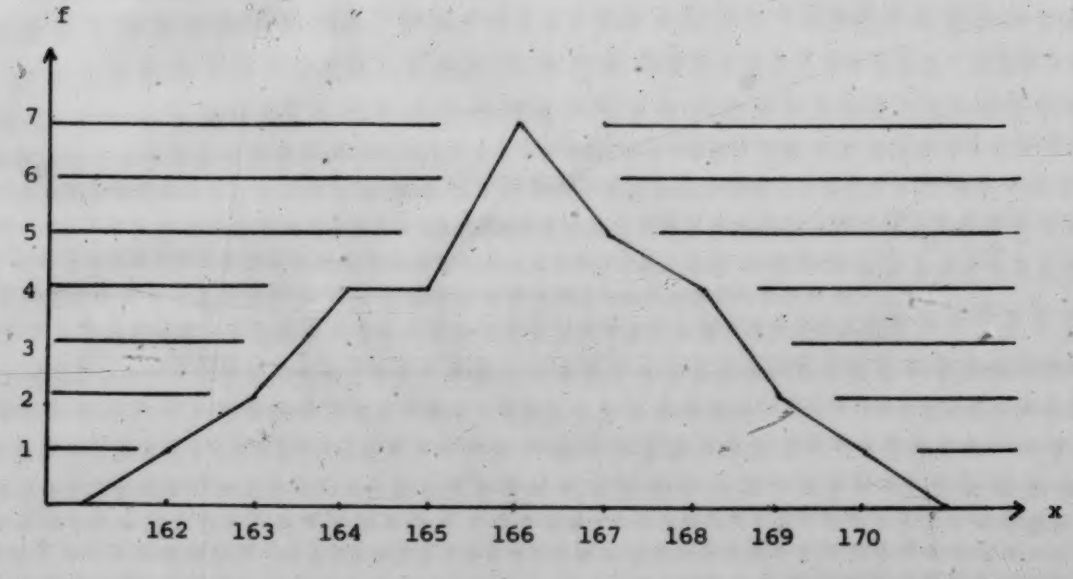
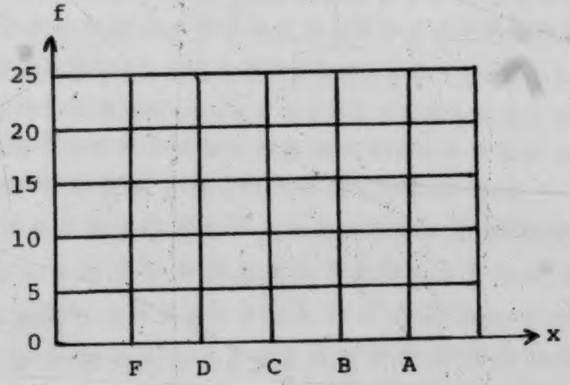


3. Construct the frequency polygons from the following tables.

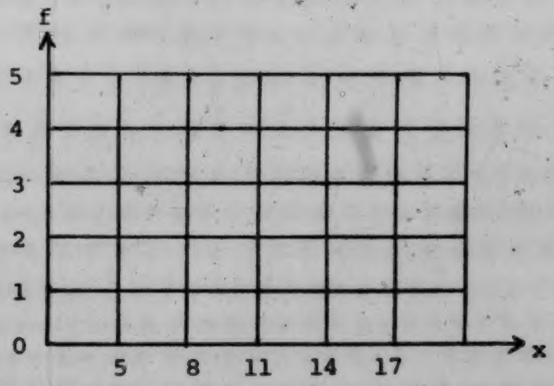
3.1

Grade	Frequency
A	5
B	15
C	25
D	10
F	5



3.2

Score	Midpoint	Frequency
16 - 18	17	1
13 - 15	14	3
10 - 12	11	5
7 - 9	8	4
4 - 6	5	2



4. 40 grade nine students have been weighed (in kilograms) with the results:

70, 73, 72, 70, 69, 69, 68, 68, 68, 67, 67, 66, 66, 66, 65, 65, 65, 65, 64, 64, 63, 63, 63, 62, 62, 62, 61, 61, 60, 60, 59, 59, 58, 57, 57, 56, 55, 54, 53, 50.

4.1 Complete the table by using the above data.

x	Tally	f
74		
73		
72		
71		
70		
69		
68		
67		
66		
65		
64		
63		
62		
61		
60		
59		
58		
57		
56		
55		
54		
53		
52		
51		
50		

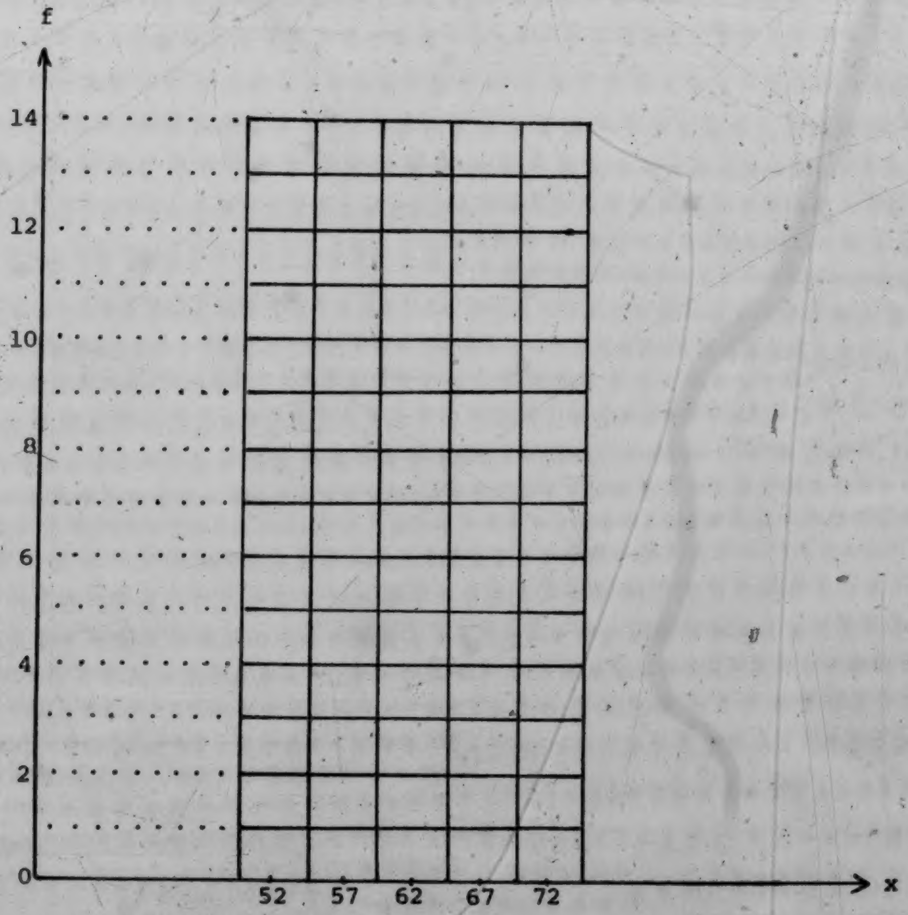
$\Sigma f = ...$

4.2 Complete the following table by using the data from 4.1.

x	Midpoint	Boundaries	f
70 - 74	72		
65 - 69	67		
60 - 64	62		
55 - 59	57		
50 - 54	52		

Σf = ...

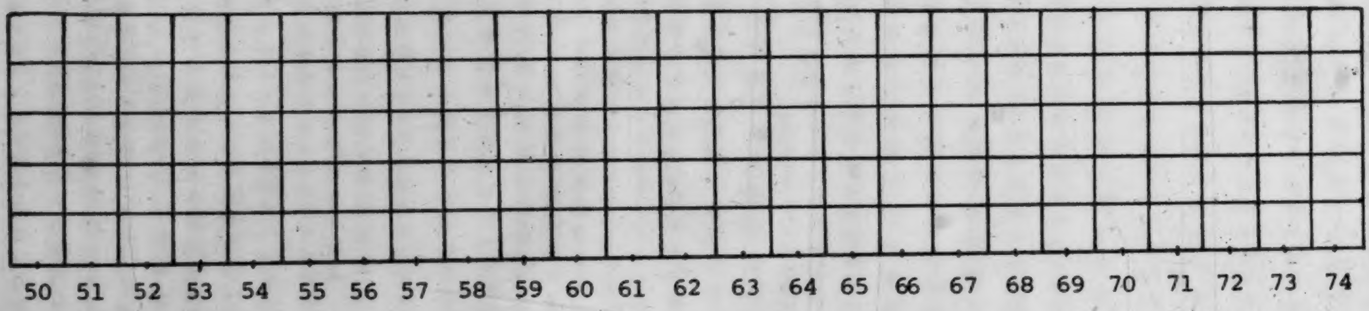
4.3 Construct the histogram for the table in 4.2.



The histogram for table 4.2

4.4 Construct the histogram for the table in 4.1

4.5 Compare the histograms for the tables in 4.1 and 4.2. Which one seems to show clearly the important feature of the distribution of the students' weight?



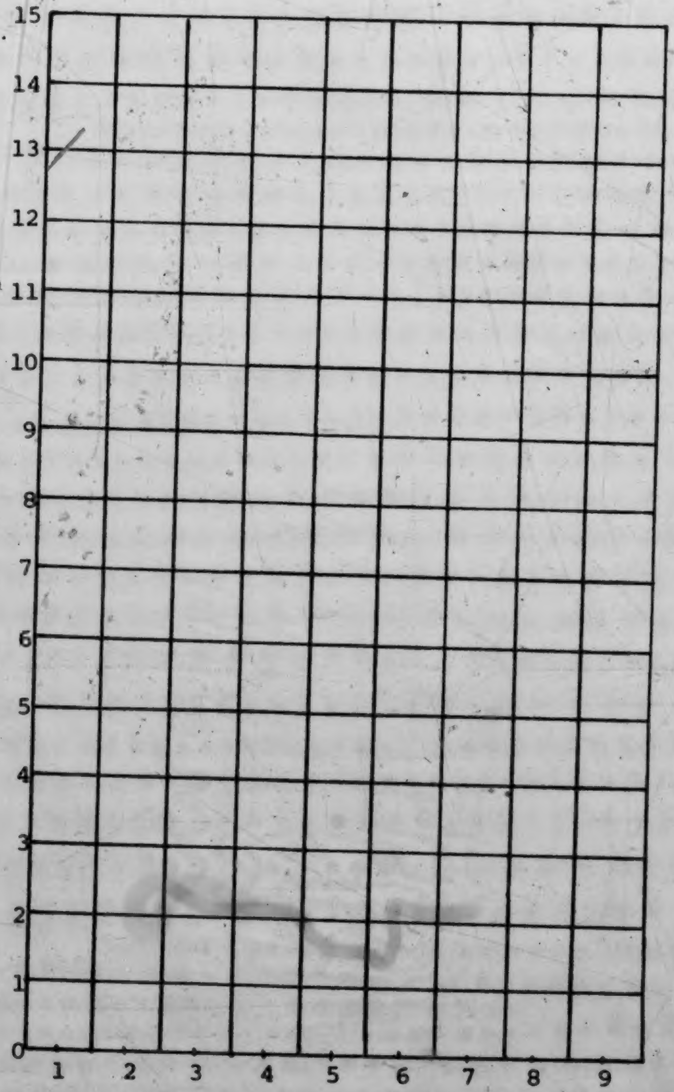
The histogram for table 4.1

LESSON 3 SHAPES OF FREQUENCY DISTRIBUTIONS

1. Instructions:
   - 1.1 Draw a histogram for every example.
   - 1.2 Observe if a histogram describes the name of a distribution.
   - 1.3 Discuss with the teacher.
2. The following are the examples:
   - 2.1 Example of a bell-shaped distribution.

Scores of the students' performance on a certain physical activity

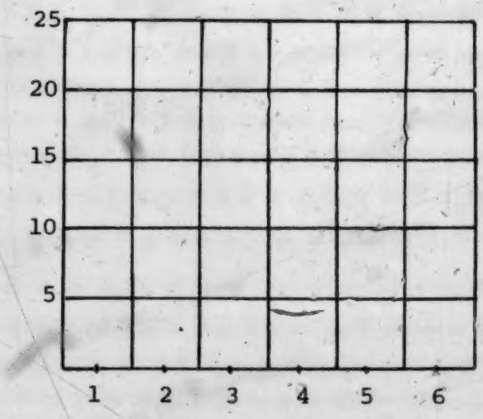
Score	No. of Students
1	1
2	3
3	5
4	10
5	15
6	10
7	5
8	3
9	1



2.2 Example of a rectangular distribution.

The number of students in a certain elementary school

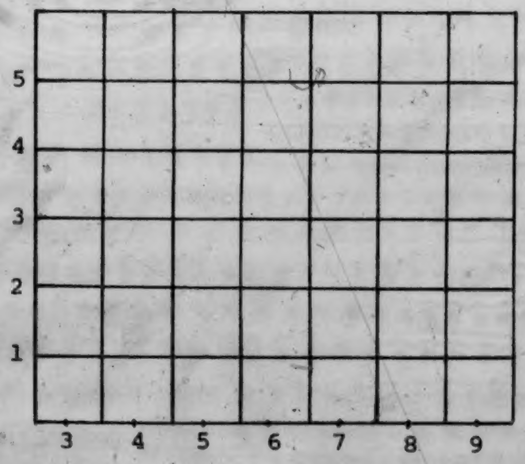
Grade	No. of Students
1	25
2	25
3	25
4	25
5	25
6	25



2.3 Example of a U-shaped distribution.

The investigation of the family size

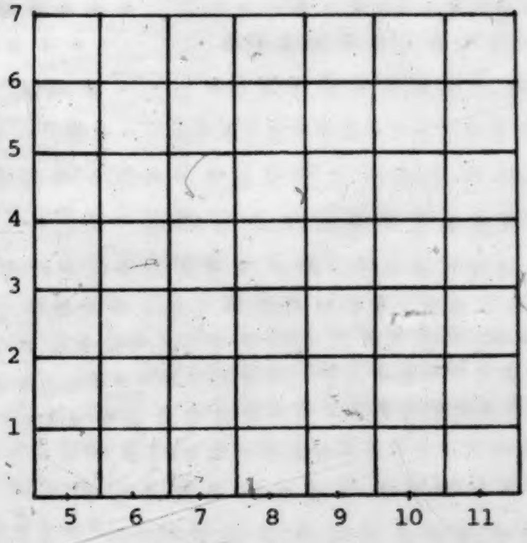
No. of People in a Family	No. of Families
3	5
4	3
5	2
6	1
7	2
8	3
9	5



2.4 Example of a triangular distribution.

The annual income of the people in a certain town

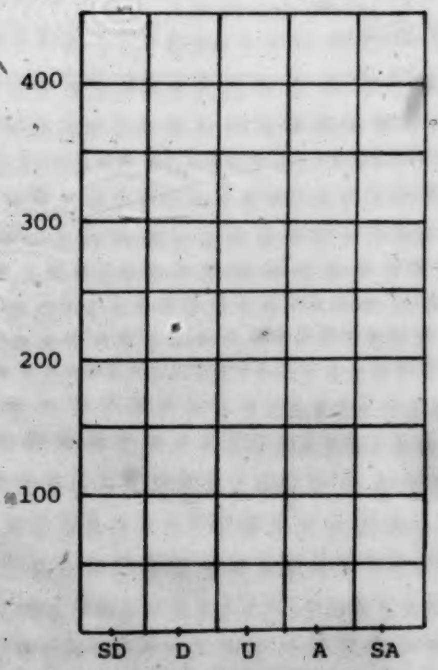
Income ($1000)	No. of People (in 1000)
5	7
6	6
7	5
8	4
9	3
10	2
11	1



2.5 Example of a miscellaneous distribution (not belonging to Example 2.1-2.4).

The opinions on the issue "Boys are allowed to have long hair"

Answer	No of Students
SD (strongly disagree)	100
D (disagree)	450
U (undecided)	300
A (agree)	200
SA (strongly agree)	100



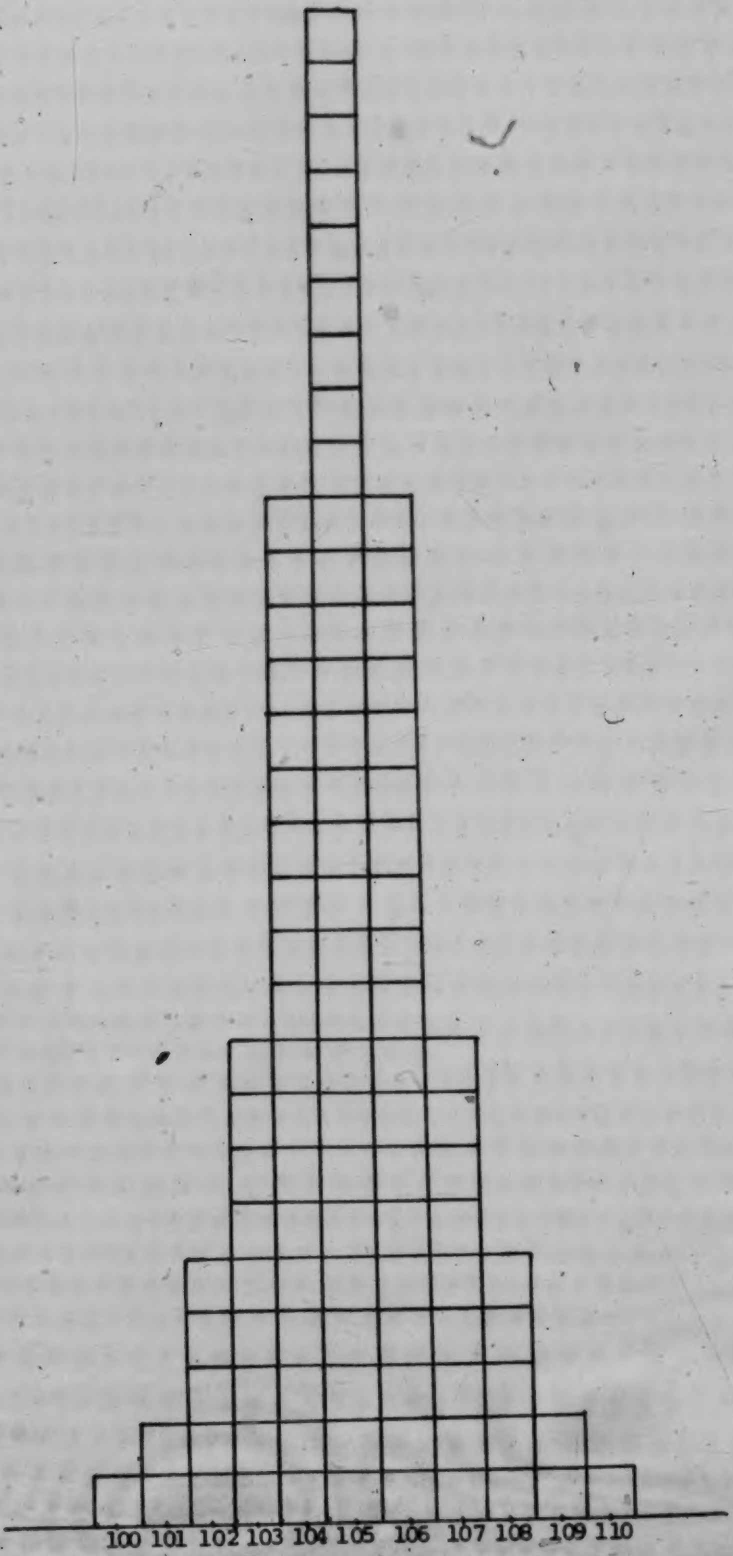
LESSON 4 EXPERIMENTAL DISTRIBUTION

1. Draw a sample of size 50 from either Box "B" or Box "T" or Box "N". Record the results in the following table.

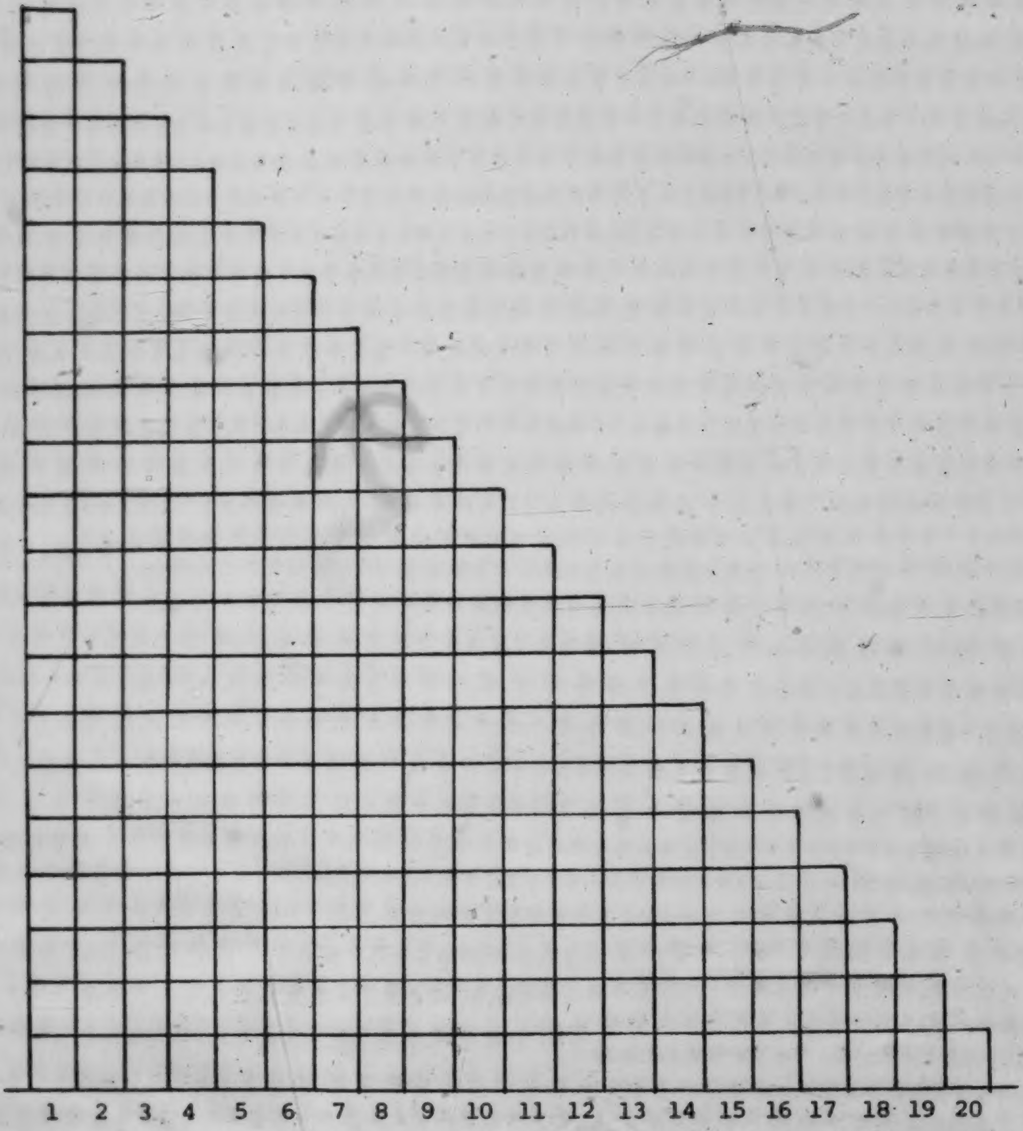
Score (x)			Tally	Frequency (f)
Box B	Box T	Box N		
100	1	1		
101	2	2		
102	3	3		
103	4	4		
104	5	5		
105	6	6		
106	7	7		
107	8	8		
108	9	9		
109	10	10		
110	11	11		
	12	12		
	13	13		
	14	14		
	15	15		
	16	16		
	17	17		
	18	18		
	19	19		
	20	20		
		21		

2. Use the data from the record sheet to construct either one of:
   2.1 A histogram
   2.2 A frequency polygon.
3. Observe the histogram or the frequency polygon in Step 2, then predict the shape of the frequency distribution in the box.

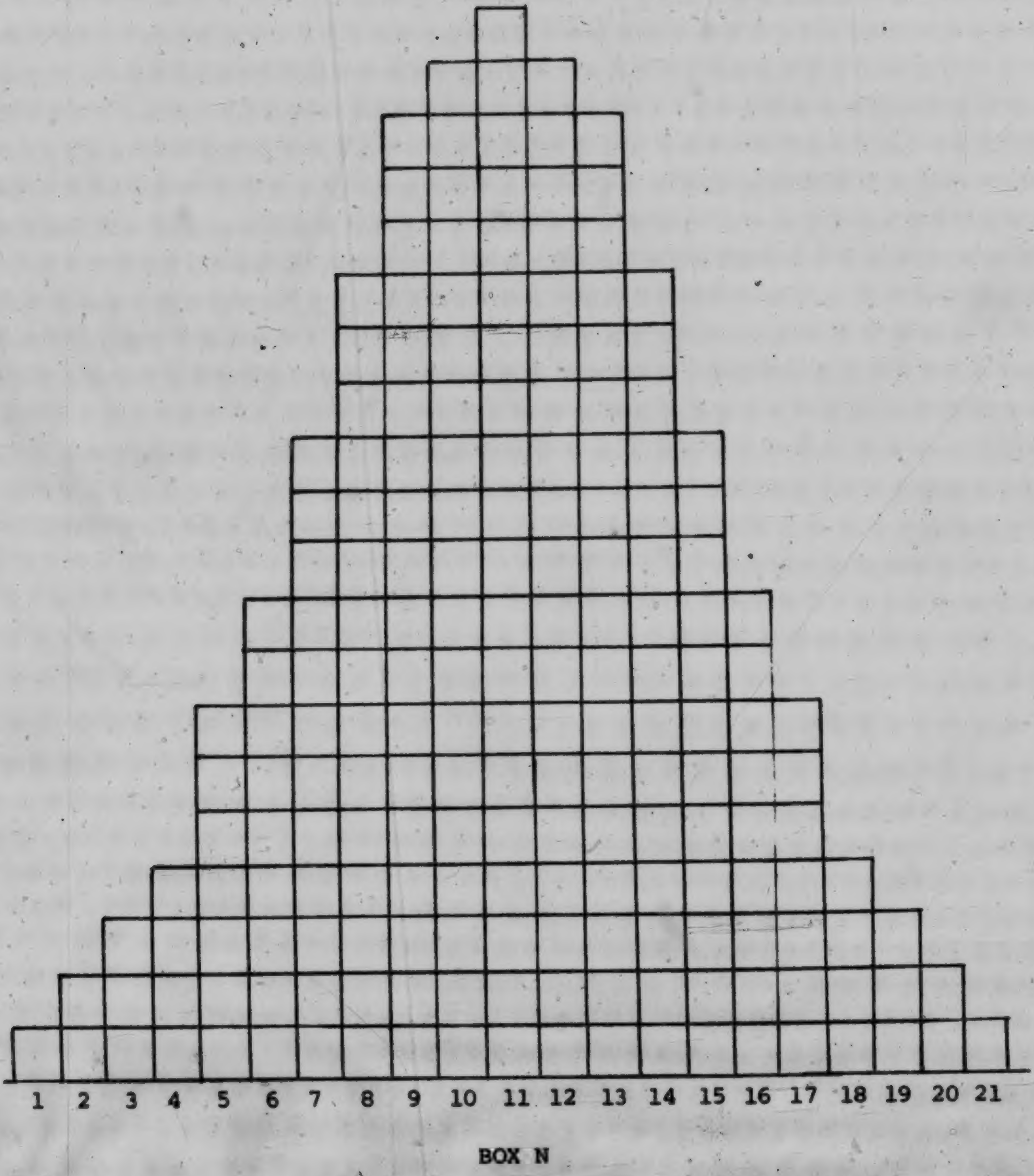
   ....



BOX B


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

BOX T


1
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12
13
14
15
16
17
18
19
20
21
BOX N

LESSON 6 MEASURE OF CENTRAL TENDENCY

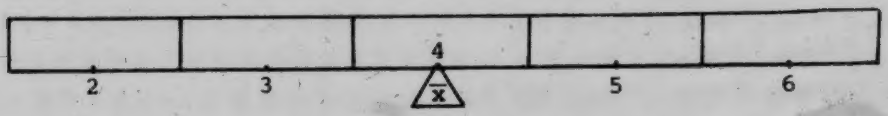
1. Mean ($\bar{x}$)

1.1 Example: A set of scores: 2, 3, 4, 5, 6

$$\text{Mean} = (2+3+4+5+5)/5 = \frac{20}{5} = 4$$

In general Mean or $\bar{x} = \frac{\ldots\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots\ldots}$

1.2 Look at the histogram of the above scores and the location of $\bar{x}$.



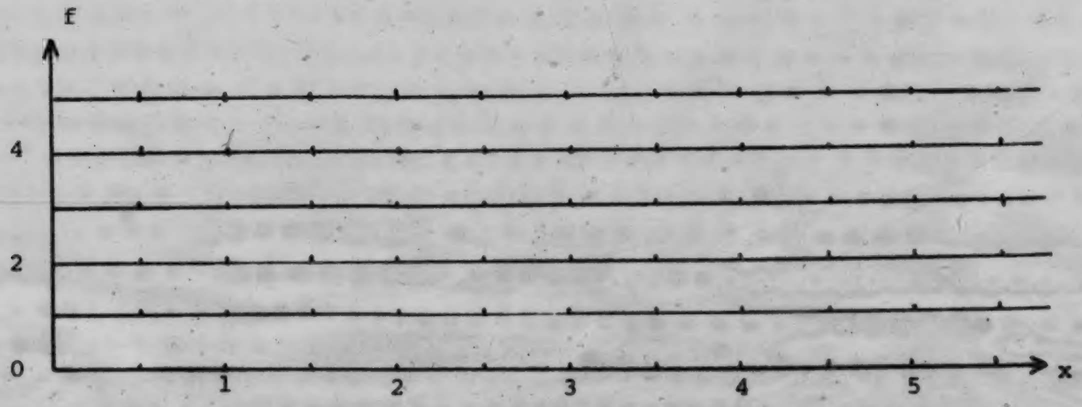
1.3 Complete the table using a set of scores: 1,2,2,3,3,3,3,4,4,5

x	f	fx

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \ldots\ldots\ldots\ldots$$

$\Sigma f = \ldots.$ $\Sigma fx = \ldots.$

1.4 Draw a histogram and mark the location of the mean.

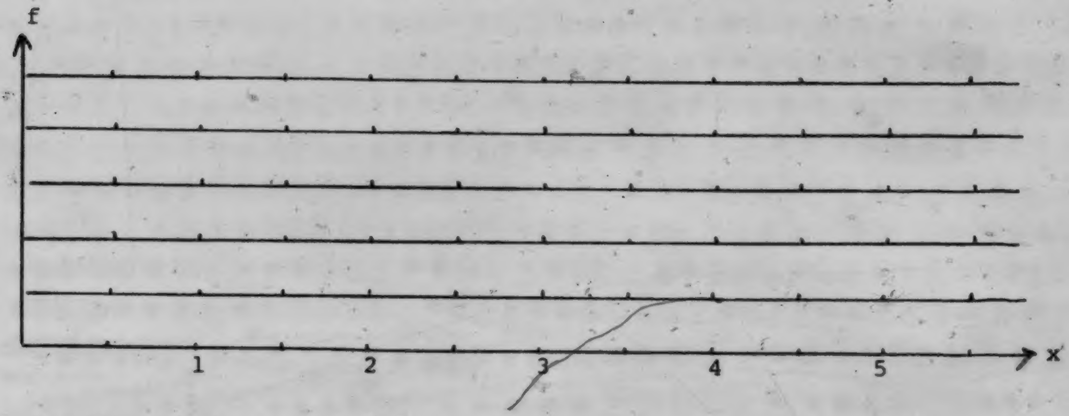


1.5 Complete the blanks

x	f	fx
5	1	
4	1	
3	2	
2	4	
1	2	
	Σf = ...	Σfx = ...

$\overline{x}$ = ...

1.6 Draw the histogram of the above table, then place a fulcrum (Δ) at the balance point of the histogram.



2. Median (Mdn)

2.1 Example: A set of scores: 2, 1, 4, 3, 5

Arrange in order: 1, 2, 3, 4, 5

Median = 3 because it is the middle score

$\overline{x}$ is also 3.

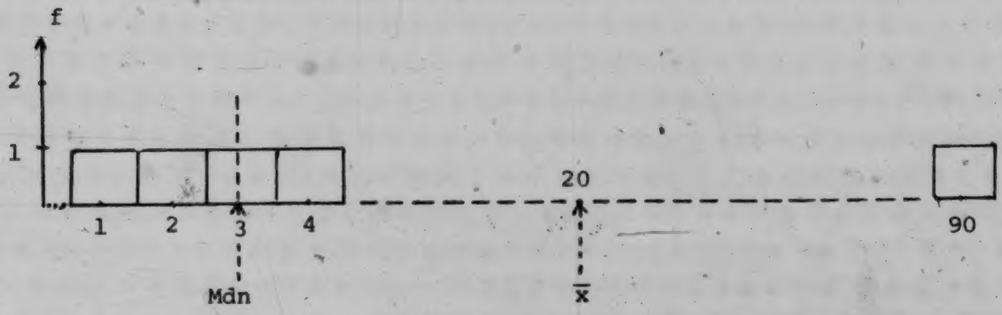
2.2 Example: A set of scores: 2, 1, 4, 3, 90

Arrange in order: 1, 2, 3, 4, 90

The median is still 3

But $\overline{x} = (1+2+3+4+90)/5 = \frac{100}{5} = 20$.

2.3 Observe the locations of mean and median of the histogram of scores: 1, 2, 3, 4, 90.



Which one (Mdn or $\overline{x}$) seems to be a better measure for the set of scores: 1, 2, 3, 4, 90?

2.4 Observe the above histogram. There are 2 1/2 blocks on either left or right of the median.

2.5 A set of scores: 2, 4, 5, 1, 70

The scores in order: ....

Mdn = ...

2.6 Example: A set of scores: 4, 2, 6, 9

Rewrite the scores in order: 2, 4, 6, 9

The middle point is between 4 and 6

which is (4+6)/2 = 5

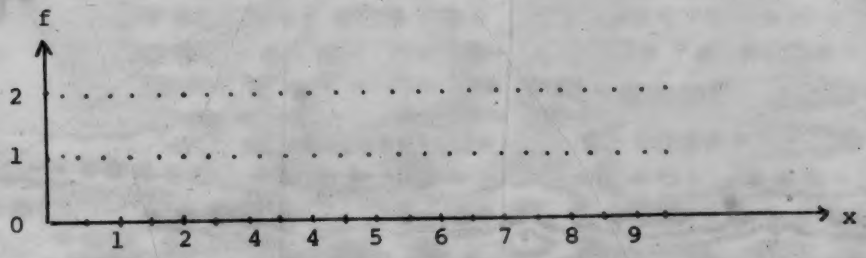
Hence Mdn = 5.

2.7 A set of scores: 13, 10, 20, 31.

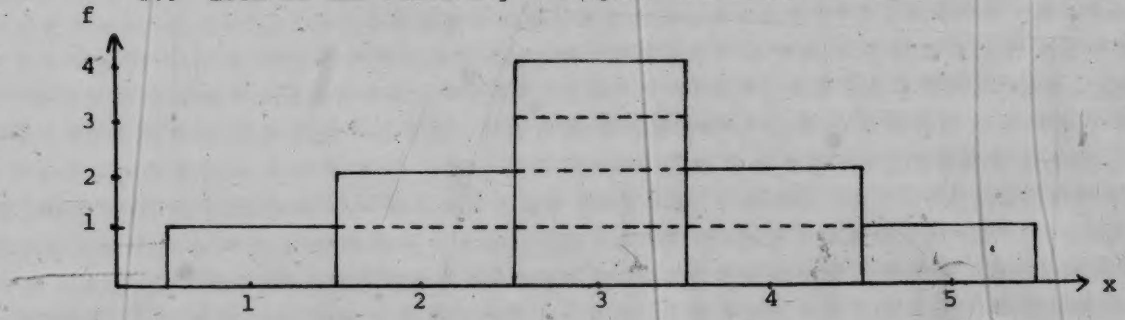
The arranging scores: ....

Mdn = ...

2.8 Draw the histogram for Example 2.6 and mark the location of mdn.



2.9 Look at the following histogram and answer the questions.



2.9.1 What is the median? Mdn = ...

2.9.2 What is the mean? $\overline{x}$ = ...

2.9.3 Does this type of distribution indicate something about $\overline{x}$ and Mdn?

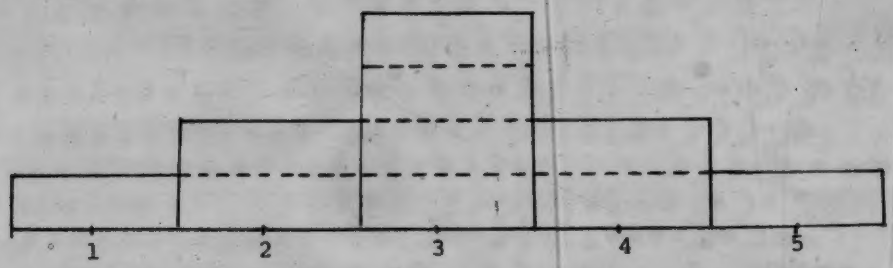
3. Mode

3.1 Example: A set of scores: 2, 3, 4, 4, 4, 5, 5
Mode is 4, because 4 has the most frequency.

3.2 Example: A set of scores: 1, 2, 3, 4, 5
There is no mode. (Why?)

3.3 Look at the following histograms and answer the questions.

3.3.1

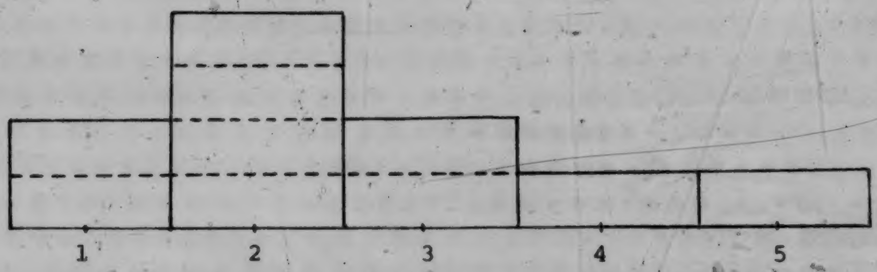


What is the mean? $\overline{x}$ = ...

What is the median? Mdn = ...

What is the mode? Mode = ...

3.3.2



The mean is 2.5

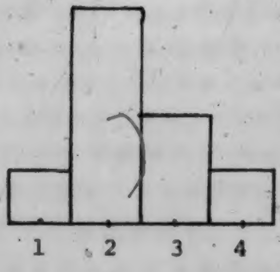
The median is 2.25

What is the mode? Mode = ...

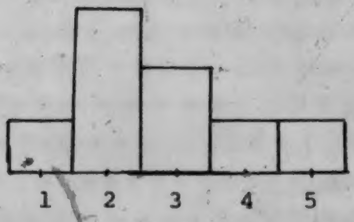
LESSON 8 VARIABILITY

1. Which one seems to be more variable than the other?

1.1



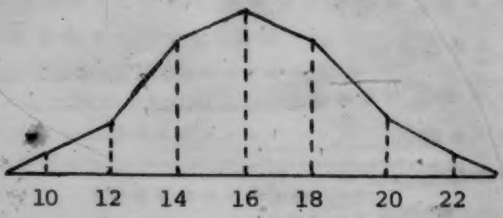
Histogram A



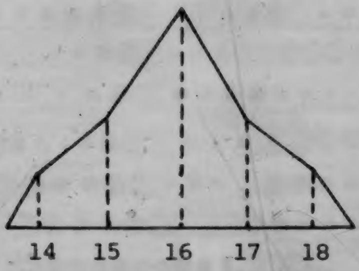
Histogram B

.... is more variable than ....

1.2



Frequency Polygon A



Frequency Polygon B

.... represents the most variable distribution.

1.3 The heights of two groups of students (in centimeters):

Group A: 145, 150, 160, 170, 175

Group B: 150, 160, 160, 160, 170

$\overline{x}$ for Group A is ....

$\overline{x}$ for Group B is ....

The most variable group is ....

RANGE

2. Find the ranges of the following scores:

2.1 Scores: 1, 3, 2, 5, 7

Range = 7 - 1 = 6

2.2 Scores: 1, 0, 3, 2, 5

Range = ...

2.3 Scores: -2, -1, 0, 1, 2

Range = ...

2.4 In general

Range = ... minus ....

STANDARD DEVIATION (SD OR S)

3. Example: Find the standard deviation of 1, 2, 3, 4, 5

First we find $\overline{x} = (1+2+3+4+5)/5 = 3$ (N = 5)

Scores x	Deviation $x - \overline{x}$	Squared Deviation $(x - \overline{x})^2$
1	$(1-3) = -2$	$(-2)^2 = 4$
2	$(2-3) = -1$	$(-1)^2 = 1$
3	$(3-3) = 0$	$0^2 = 0$
4	$(4-3) = 1$	$1^2 = 1$
5	$(5-3) = 2$	$2^2 = 4$
	$\Sigma(x-\overline{x}) = 0$	$\Sigma(x-\overline{x})^2 = 4+1+0+1+4 = 10$

$$SD^2 = \frac{\Sigma(x-\overline{x})^2}{N} = \frac{10}{5} = 2$$

$$SD = \sqrt{2} = 1.414$$

3.1 Discuss the steps in calculating the standard deviation with the teacher.

3.2 Complete the following:

Standard Deviation = The square root of the average of

....

4. Example: Find the standard deviation from the table of frequency distribution.

x	f	fx	d or $(x-\overline{x})$	d^2	fd^2
5	1	5	2	4	1x4 = 4
4	2	8	1	1	2x1 = 2
3	4	12	0	0	4x0 = 0
2	2	4	-1	1	2x1 = 2
1	1	1	-2	4	1x4 = 4
	$\Sigma f = 10$	$\Sigma fx = 30$			$\Sigma fd^2 = 12$

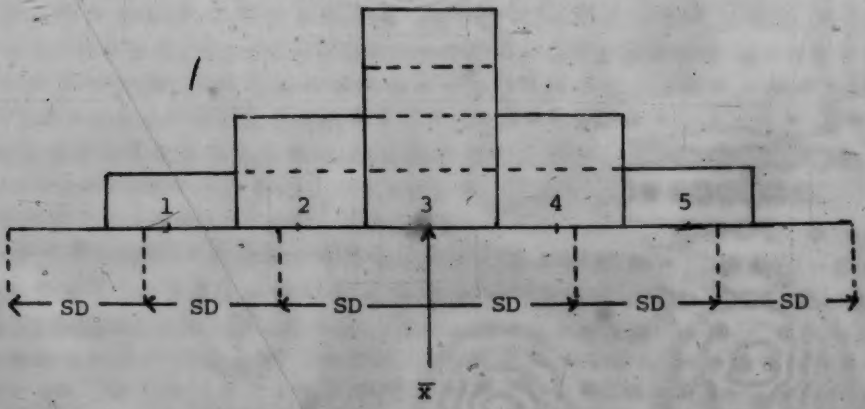
$$\overline{x} = \frac{\Sigma fx}{\Sigma f} = \frac{30}{10} = 3$$

$$SD^2 = \frac{\Sigma fd^2}{N} = \frac{12}{10} = 1.2$$

$$SD = \sqrt{1.2} = 1.1 \text{ (approx.)}$$

4.1 Discuss the above example with the teacher.

4.2 Observe the histogram and how SD is used to measure the spread.



5. There are three groups of students whose scores in a mathematics test were as follows:

Group A x	Group A f	Group B x	Group B f	Group C x	Group C f
80	1	80	1	80	1
70	1	60	1	70	1
65	1	58	1	55	6
60	1	56	1	40	1
55	2	55	2	30	1
50	1	54	1		
45	1	52	1		
40	1	50	1		
30	1	30	1		

5.1 Find the means.

$\overline{x}$ of Group A = ...

$\overline{x}$ of Group B = ...

$\overline{x}$ of Group C = ...

5.2 Find the ranges.

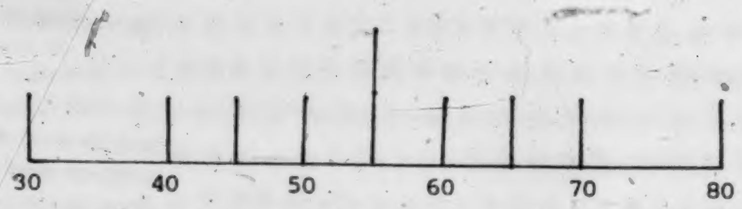
The range of Group A = ...

The range of Group B = ...

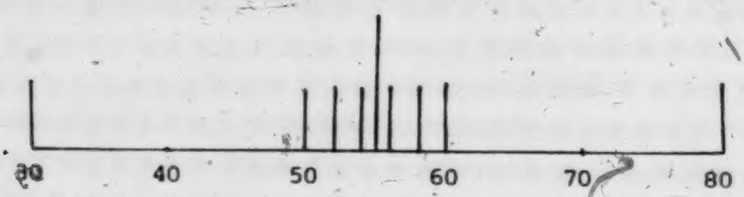
The range of Group C = ...

5.3 Do the results obtained in 5.1 and 5.2 indicate the differences among the three groups? Why?

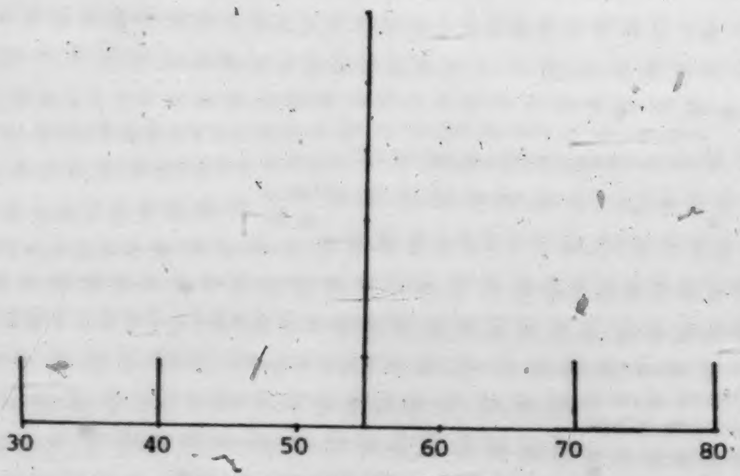
5.4 Look at the graphs of the three groups:



Group A



Group B



Group C

The most variable group is ....

The least variable group is ....

5.5 Find the standard deviations:

SD for Group A is 14 (approx.)

SD for Group C is 13 (approx.)

What is SD for Group B? ....

5.6 Compare the results of 5.4 with 5.5.

LESSON 10 TRANSFORMED SCORES

1. Part A. If a set of scores is transformed to another set by adding or subtracting a constant.

1.1 Look at the following tables:

x	f
1	3
2	5
3	1
4	1
	N = 10

Table 1

x'	f
6	3
7	5
8	1
9	1
	N = 10

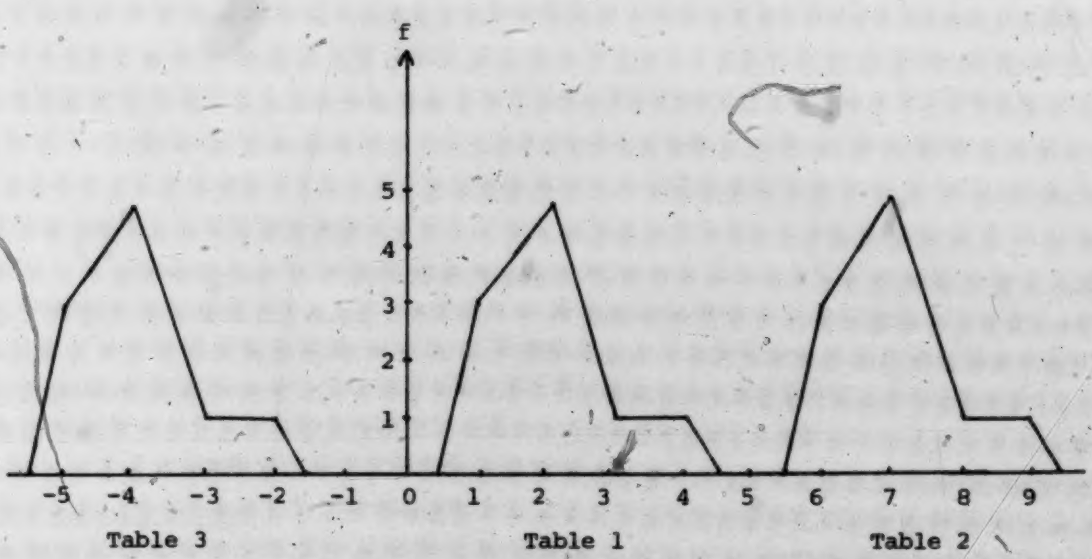
Table 2

x"	f
-5	3
-4	5
-3	1
-2	1
	N = 10

Table 3

1.2 Observe how Table 2 and Table 3 are transformed from Table 1.

1.3 Look at the frequency polygons of the three tables.



1.4 Look at the computation for $\overline{x}$ and SD of the three tables:

x	f	fx	d $(x-\overline{x})$	d^2	fd^2
1	3	3	-1	1	3
2	5	10	0	0	0
3	1	3	1	1	1
4	1	4	2	4	4
	$\Sigma f=10$	$\Sigma fx=20$			$\Sigma fd^2=8$

$$\overline{x}_1 = \frac{\Sigma fx}{\Sigma f} = \frac{20}{10} = 2$$

$$SD_1^{\ 2} = \frac{\Sigma fd^2}{\Sigma f} = \frac{8}{10} = .8$$

$$SD_1 = \sqrt{.8} = .9 \text{ (approx.)}$$

x'	f	fx'	d	d^2	fd^2
6	3	18	-1	1	3
7	5	35	0	0	0
8	1	8	1	1	1
9	1	9	2	4	4
	10	70			8

$$\overline{x}_2 = \frac{70}{10} = 7$$

$$SD_2^{\ 2} = \frac{8}{10} = .8$$

$$SD_2 = \sqrt{.8} = .9$$

x"	f	fx"	d^2	fd^2
-5	3	-15	1	3
-4	5	-20	0	0
-3	1	-3	1	1
-2	1	-2	4	4
	10	-40		8

$$\overline{x}_3 = \frac{-40}{10} = -4$$

$$SD_3^{\ 2} = \frac{8}{10} = .8$$

$$SD_3 = \sqrt{.8} = .9$$

1.5 Observe the means and standard deviations of the three tables.

1.6 Conclusion:

If a new set of scores is received from an old set of scores by adding a given number then

a. The mean of a new set is equal ....

b. The standard deviations of the two sets are ....

2. Part B. If a set of scores is transformed to another set by multiplying or dividing with a constant.

2.1 Look at the following tables:

x	f
1	3
2	5
3	1
4	1
	N = 10

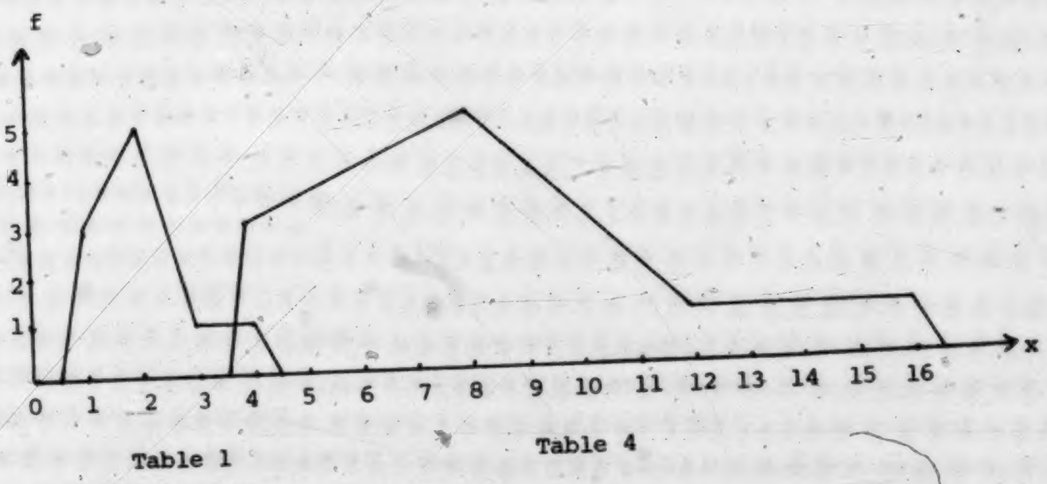
Table 1

x'''	f
4	3
8	5
12	1
16	1
	N = 10

Table 4

2.2 Observe how Table 4 is transformed from Table 1.

2.3 Look at the frequency polygons of Table 1 and Table 4.



2.4 Look at the computation for $\bar{x}$ and SD of Table 1 and Table 4.

2.4.1 Table 1 has already been computed:

$$\bar{x}_1 = 2$$

$$SD_1^{\,2} = .8$$

$$SD_1 = .9$$

2.4.2 For Table 4:

x'''	f	fx'''	d	fd	fd²
4	3	12	-4	-12	48
8	5	40	0	0	0
12	1	12	4	4	16
16	1	16	8	8	64
	N=10	Σfx'''=80			Σfd²=128

$$\bar{x}_4 = \frac{80}{10} = 8$$

$$SD_4^{\,2} = \frac{128}{10} = 12.8$$

$$SD_4 = \sqrt{12.8} = 3.6$$

2.5 Observe the results:

Table 4 = Table 1 multiplied by 4.

$\bar{x}_4(8) = \bar{x}_1(2)$ multiplied by 4.

$SD_4(3.6) = SD_1(.9)$ multiplied by 4.

2.6 Conclusion:

If a new set of scores is received from an old set of scores by multiplying with a given number then

a. the mean of the new set is equal to the mean of the old set .... that given number.

b. the standard deviation of the new set is equal

....

3. Part C. Standard scores.

3.1 Look at Table 5 which is transformed from Table 1 by multiplying by 5.

x	f
5	3
10	5
15	1
20	1
	N=10

$\bar{x}_5 = 10$

$SD_5 = 4$

Table 5

3.2 Table 5 can be transformed to Table 6 by subtracting $\bar{x}_5$(10) or the mean of Table 5 as the result.

x	f
-5	3
0	5
5	1
10	1
	N=10

$\bar{x}_6 = 0$

$SD_6 = 4$

Table 6

3.3 Table 6 can be transformed to Table 7 by dividing with SD_6(4) or the standard deviation of Table 6 as the result.

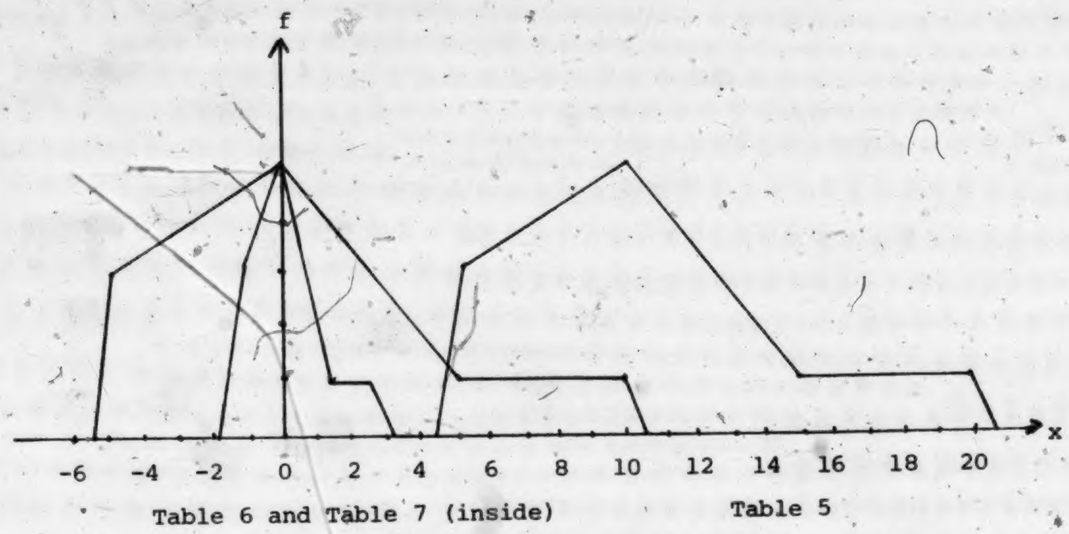
x	f
-1.25	3
0	5
1.25	1
2.5	1
	N=10

$\bar{x}_7 = 0$

$SD_7 = 1$

Table 7

3.4 Look at the frequency polygons of Table 5, 6, and 7.



3.5 The method that we use to transform Table 5 to Table 7 is called "to standardize scores or data into standard score." We can combine the two steps (in 3.1 to 3.3); we will get the formula:

$$\text{A standard score} = \frac{\text{A score - the mean}}{\text{The standard deviation}}$$

$$\text{or} \qquad z = \frac{x-\bar{x}}{SD}$$

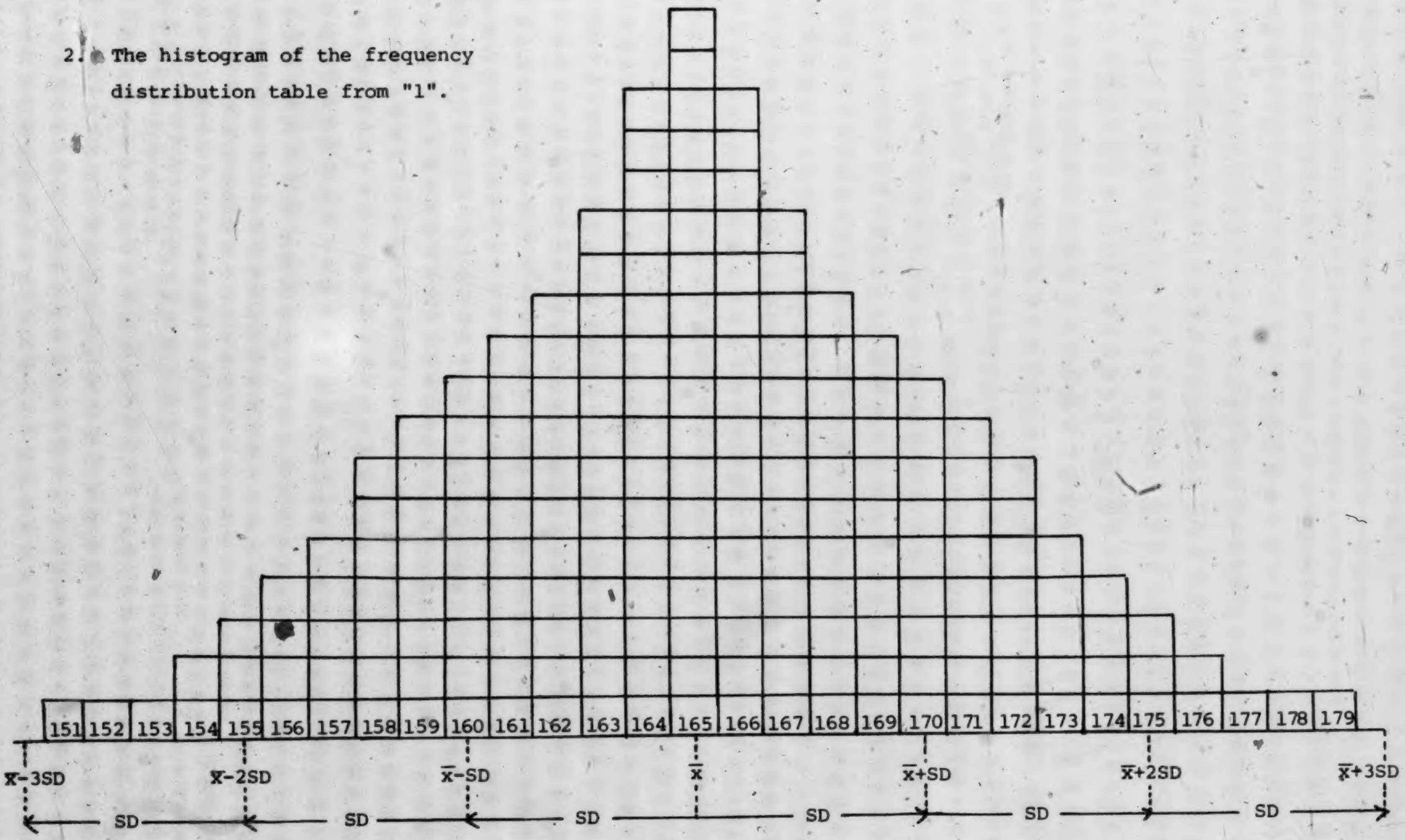
LESSON 12 NORMAL DISTRIBUTION

1. Given the frequency distribution table of the heights (in centimeters of 200 people:

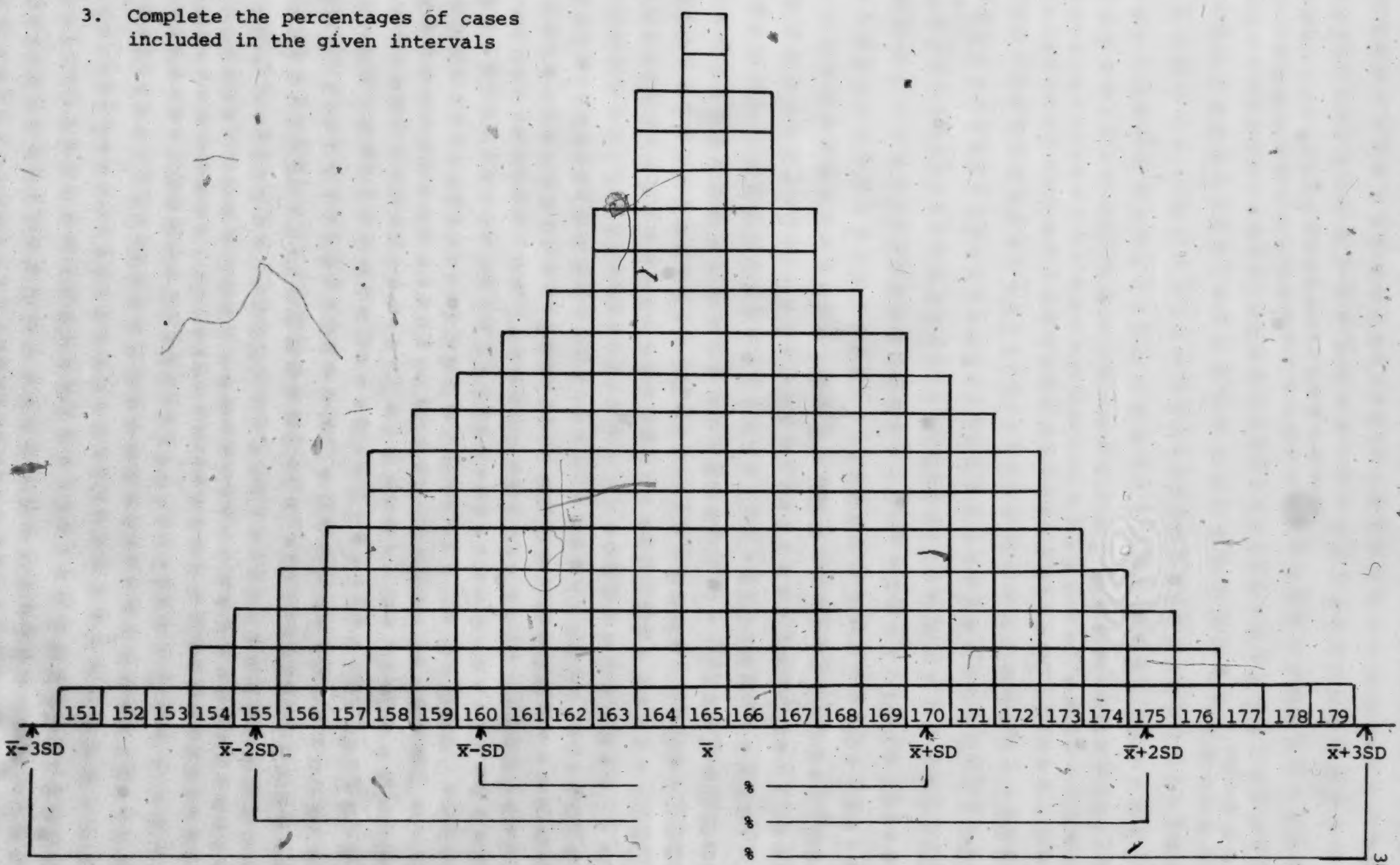
x	f	x	f	x	f
151	1	165	18	166	16
152	1			167	13
153	1			168	11
154	2			169	10
155	3			170	9
156	4			171	8
157	5			172	7
158	7			173	5
159	8			174	4
160	9			175	3
161	10			176	2
162	11			177	1
163	13			178	1
164	16			179	1

$\Sigma f=200$

2. The histogram of the frequency distribution table from "1".

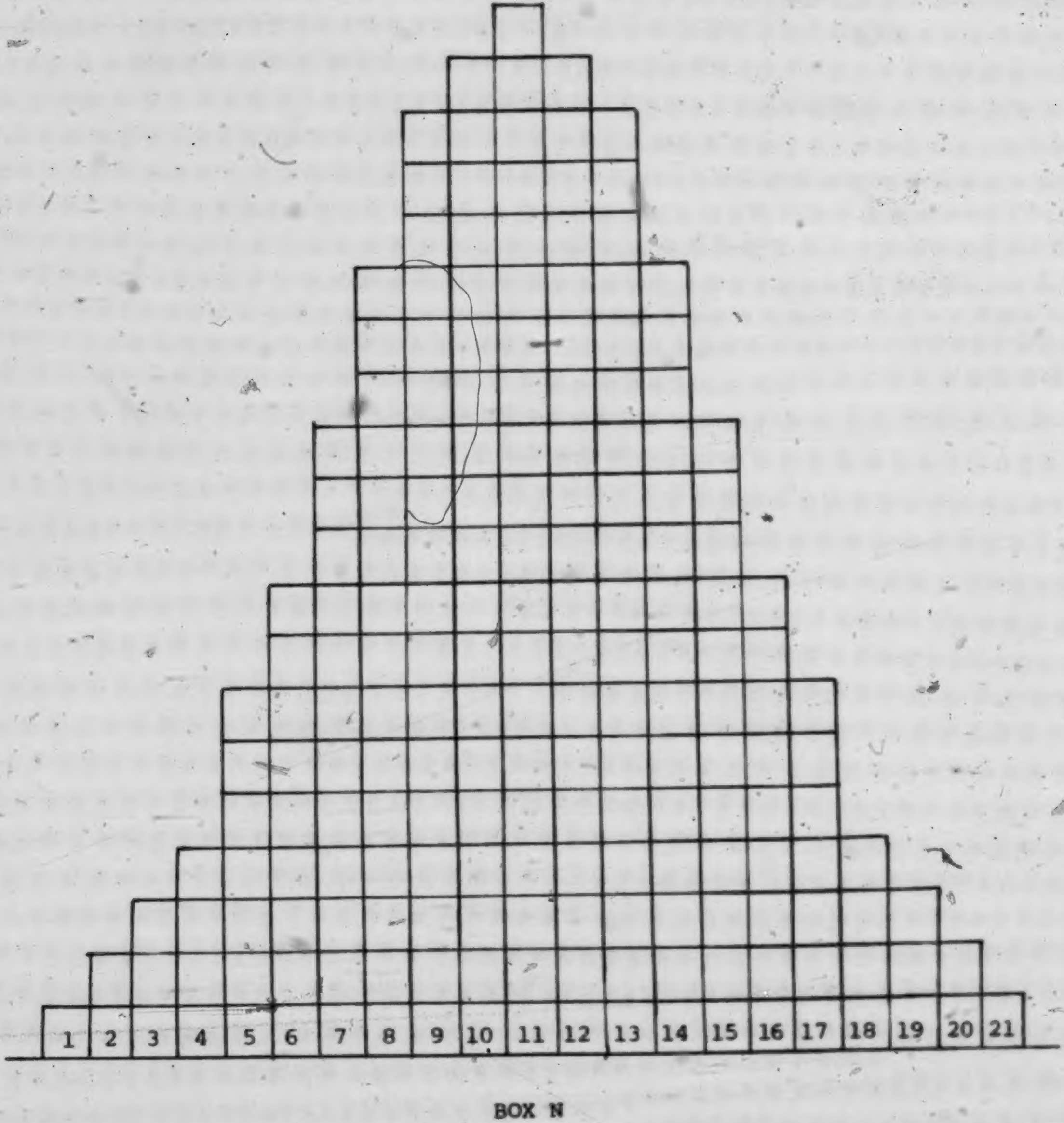


3. Complete the percentages of cases included in the given intervals



LESSON 13 DISTRIBUTION OF SAMPLE MEANS

1. Draw 10 random samples of size 2 "with replacement" from Box N, then record the result in "Recording Sheet 1."
2. Complete "Recording Sheet 2" by recording first 10 means ($\overline{x}$) from your own "Recording Sheet 1," and another 90 means from any of your nine different classmates.
3. Draw the histogram for the 100 means recorded in "Recording Sheet 2."
4. Discuss the concepts with the teacher (use the blank below to note).


BOX N

RECORDING SHEET 1

SAMPLE OF SIZE

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	$\bar{x}$
Example 1										/	/											10.5
Example 2						/	/		/		/	/										9
Sample 1																						
Sample 2																						
Sample 3																						
Sample 4																						
Sample 5																						
Sample 6																						
Sample 7																						
Sample 8																						
Sample 9																						
Sample 10																						

RECORDING SHEET 2

LESSON 13, 14

$\bar{x}$	Tally	f	$\bar{x}$	Tally	f	$\bar{x}$	Tally	f	$\bar{x}$	Tally	f
1			6			11			16		
1.2			6.2			11.2			16.2		
1.4			6.4			11.4			16.4		
1.5			6.5			11.5			16.5		
1.6			6.6			11.6			16.6		
1.8			6.8			11.8			16.8		
2			7			12			17		
2.2			7.2			12.2			17.2		
2.4			7.4			12.4			17.4		
2.5			7.5			12.5			17.5		
2.6			7.6			12.6			17.6		
2.8			7.8			12.8			17.8		
3			8			13			18		
3.2			8.2			13.2			18.2		
3.4			8.4			13.4			18.4		
3.5			8.5			13.5			18.5		
3.6			8.6			13.6			18.6		
3.8			8.8			13.8			18.8		
4			9			14			19		
4.2			9.2			14.2			19.2		
4.4			9.4			14.4			19.4		
4.5			9.5			14.5			19.5		
4.6			9.6			14.6			19.6		
4.8			9.8			14.8			19.8		
5			10			15			20		
5.2			10.2			15.2			20.2		
5.4			10.4			15.4			20.4		
5.5			10.5			15.5			20.5		
5.6			10.6			15.6			20.6		
5.8			10.8			15.8			20.8		
									21		

APPENDIX C

ACHIEVEMENT TEST

NAME ....

ACHIEVEMENT TEST

PART I

INSTRUCTION

Place your answers or solutions in the spaces (___________) which are provided for each item.

Use the provided spaces to work some items.

1. The scores for a mathematics test in a class of twenty-nine students were as follows:

8, 6, 7, 6, 4, 5, 5, 7, 6, 7, 7, 5, 7, 9,
7, 8, 6, 8, 4, 6, 7, 5, 5, 7, 8, 5, 5, 8, 9

1.1 What is the total frequency in the above set of scores? 1.1 (___________)

1.2 What is the frequency of the score 5? 1.2 (___________)

1.3 What is the frequency of the score 7? 1.3 (___________)

2. The weights in kilogram of 30 students were as follows:

67	41	49	59	51	55	59	58	49	54
46	62	48	63	47	63	43	56	45	67
60	42	61	47	57	43	53	42	44	58

The students' weights are grouped into the class intervals:

40-44; 45-49; 50-54; 55-59; 60-64; 65-69

2.1 What is the frequency corresponding to the 45-49 class interval? 2.1 (___________)

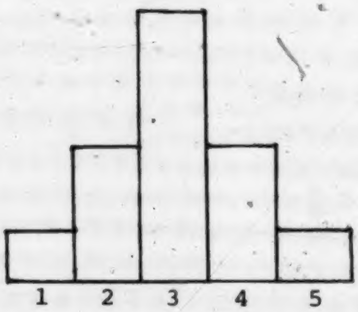
2.2 What is the frequency corresponding to the 60-64 class interval? 2.2 (___________)

2.3 What are the boundaries of the 50-54 class interval? 2.3 (___________)

2.4 What is the width of the 40-44 class interval? 2.4 (___________)

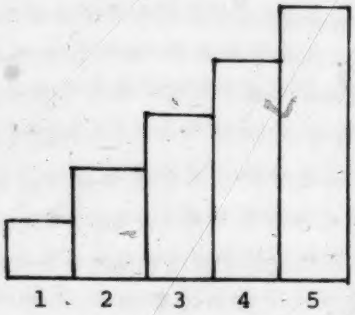
3. For each of the distributions below, select a word that best describes its shape from the following list:
U-shaped, Rectangular, Triangular, Bell-shaped, and Miscellaneous

3.1



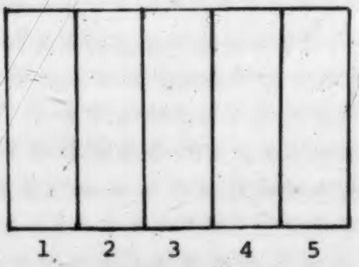
3.1 (__________)

3.2



3.2 (__________)

3.3



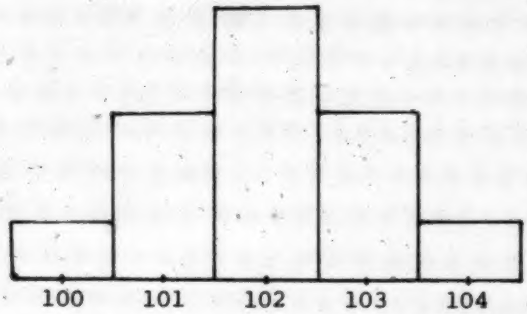
3.3 (__________)

4. What is the mean of the scores: 2, 1, 4, 7, 4, 12?
(use the provided space below to work.) 4. (__________)

5. What is the median of the scores: 17, 12, 20, 15, 25?
5. (__________)

6. What is the mode of the scores: 2, 1, 3, 2, 5, 7, 3, 2, 7?
6. (__________)

7. Given the following distribution



7.1 What is the mean? 7.1 (__________)
7.2 What is the median? 7.2 (__________)
7.3 What is the mode? 7.3 (__________)

8. What is the range of the scores
2, 1, 7, 3, 2, 5, 7, 3, 2? 8. (__________)

9. "At noon, March 25, 1976, the temperature was 6°C. The record high temperature for this day was 13°C in 1940. The record low temperature for this day was -30°C in 1911."
What is the range of the temperature for March 25 since 1900?
9. (__________)

10. Find the standard deviation of the scores: 1, 3, 5, 7, 4.
(use the provided space below to work.) 10. (__________)

11. Set A is a given set of numbers which has a mean of 12 and a standard deviation of 4:

11.1 If Set B is obtained from Set A by adding 90 to each number in Set A

11.1a What is the mean for Set B? 11.1a (__________)

11.1b What is the standard deviation for Set B? 11.1b (__________)

11.2 If Set C is obtained from Set A by multiplying each number in Set A by 2

11.2a What is the mean for Set C? 11.2a (__________)

11.2b What is the standard deviation for Set C? 11.2b (__________)

11.3 If Set D is obtained from Set A by subtracting 12 from each number in Set A and then dividing each result by 4

11.3a What is the mean for Set D? 11.3a (__________)

11.3b What is the standard deviation for Set D? 11.3b (__________)

12. Suppose a population consists of (10, 12, 30, 14, 50)

12.1 A sample of size 2 is taken from the above population without replacement. Write four different possibilities for this sample.

12.1 [(____,____) (____,____) (____,____) (____,____)]

12.2 A sample of size 4 is taken from the above population without replacement. Write two different possibilities for this sample.

12.2 [(____,____,____,____) (____,____,____,____)]

12.3 A sample of size 7 is taken from the above population with replacement. Write any one possibility for this sample.

12.3 [(____,____,____,____,____,____,____)]

13. 100 students out of the total of 1,000 students in a certain school were randomly selected to give their opinions on the issue "Should the students in this school wear uniforms?"

From the above story

13.1 What is the sample? 13.1 (__________)

13.2 What is the population? 13.2 (__________)

14. At a certain hospital the weights of new born infants have been recorded and found to have a normal distribution with a mean of 7.1 pounds and a standard deviation equal to 1 pound.

14.1 What percent of the newborn infants have weights between 6.1 and 8.1 pounds? 14.1 (__________)

14.2 Suppose there are 1,000 newborn infants in that hospital. How many of them are likely to weigh between 5.1 and 9.1 pounds? 14.2 (__________)

15. The weights of all grade nine boys in Alberta is approximately normally distributed with a mean equal to 60 kilograms:

15.1 Suppose 100 samples of size 30 were taken from the above population, and for each sample the mean weight is computed. In all there would be 100 means for the size 30 samples.

15.1a What would the distribution of the 100 means look like? 15.1a (__________)

15.1b Estimate the mean for the above 100 means. 15.1b (__________)

15.2 Suppose 100 samples of size 50 were taken from the above population, and for each sample the mean weight is computed. In all there would be 100 means for the size 50 samples.

15.2a What would the distribution of the 100 means look like? 15.2a (__________)

15.2b Estimate the mean for the above 100 means? 15.2b (__________)

15.3 How would the distributions in 15.1a and 15.2a differ?

15.3 (____)

15.4 If you were to compute the means for the distributions in 15.1a and 15.2a, which one would likely be closer to 60 (60 is the population mean)? 15.4 (__________)

16. The marks of twenty students were randomly selected from the marks of all grade six students in a certain elementary school. This was done to gather evidence about grade six marks before students enter grade seven. The following were the twenty marks (in percent):

60	62	65	70	55	75	73	78	80	90
55	85	81	74	58	68	70	75	74	72

16.1 What is the population from which the sample was taken? 16.1 (__________)

16.2 What is the sample? 16.2 (__________)

16.3 What is the mean of the sample? (use the provided space below to work.) 16.3 (__________)

16.4 Estimate the average grade six mark for the students before starting grade seven. 16.4 (__________)

17. Two students, A and B, would like to estimate the annual income of the taxi drivers in Edmonton.
Student A asked eight randomly chosen taxi drivers and received the following replies: 7000, 8000, 5000, 9000, 6000, 8000, 6000, 7000.
Student B independently asked twelve randomly chosen taxi drivers and received the following replies: 5000, 8000, 12000, 11000, 6000, 9000, 8000, 9000, 7000, 9000, 10000, 8000.

17.1 What is the annual income estimated by Student A? 17.1 (____________)

17.2 What is the annual income estimated by Student B? 17.2 (____________)

17.3 Generally which of the two students would have the better estimate? 17.3 (____________)

(use the provided space here to work for 17.1 and 17.2)

PART II

INSTRUCTION

Answer the following questions briefly by writing your answers within the provided spaces.

18. Suppose you are assigned to organize and describe by some statistical procedures the mathematics scores for a group of one hundred grade nine students.

18.1 What would you do so that you could describe the distribution of the scores?

18.1 (____
____)

18.2 What would you do so that you could get a single score that would seem to represent the whole group?

18.2 (____
____)

18.3 What would you do so that you could get a number that would indicate the spread of the scores?

18.3 (____
____)

19. Suppose you are given a box containing 1000 buttons with a number written on each of them. You are told that the distribution of those numbers is normal. And you are allowed to pick a sample of any size from the box with replacement.

19.1 What evidence would you take from the sample to support the statement that the distribution of numbers in the box is normal?

19.1 (____

____)

19.2 What would you do to estimate the mean of the numbers in the box?

19.2 (____

____)

20. Suppose you are asked to find out what the most popular, the second-most popular, and the third most-popular songs are among junior high students in Edmonton during the month of June 1976. Explain your procedure step by step.

21. Suppose you are asked to make a comparison of the heights of grade nine students in Calgary with the heights of grade nine students in Edmonton.

Explain your procedure step by step.

22. Suppose you are asked to predict the outcome of the election among three candidates A, B, and C in a certain region. Explain your procedure step by step.

.... END OF TEST ....

APPENDIX D

CONCEPTION TEST

NAME ....

CONCEPTION TEST

INSTRUCTION: ANSWER THE FOLLOWING QUESTIONS, AS IF YOU ARE BEING INTERVIEWED, IN THE PROVIDED SPACES.

1. The people in your mathematics class are each weighed and the weights are recorded. Your friend, who is not in the class, wants to know what the "middle" weight is? How would you answer this question?

2. The "mean" of a set of numbers is another name for the "average" of the set. For example, the mean of the set of numbers (2, 2, 3, 9, 10, 10) is six because the sum of the numbers divided by six is equal to six. If 123 is added to each of the numbers then the set consists of (125, 125, 126, 132, 133, 133). What is the mean of this set of numbers? How do you get the answer?

3. The mean of the set of numbers (2, 2, 3, 9, 10, 10) is six. If each of the numbers in this set is multiplied by seven the set of numbers becomes (14, 14, 21, 63, 70, 70). What is the mean of this set? How do you get the answer?

4. If the heights of all grade nine girls in Alberta were tabulated there would be a small number that were very tall and a small number that were very short. Most of the heights would be somewhere in the middle. Such an arrangement of numbers is known as a "normal distribution." What are some other examples of "normal distributions"?

5. If 100 grade nine girls were picked from the schools of Alberta and their "mean height" was computed, would it be the same as the "mean height" of all grade nine girls in Alberta? Why or why not?

6. The mean for each of the following sets of numbers is six.

   A = (2, 2, 3, 9, 10, 10)
   B = (2, 5, 6, 6, 7, 10)
   C = (2, 5, 5, 7, 7, 10)

   A special number can be computed for set A as follows:

   1. Subtract each number in the set from the mean of the set. For Set A subtract each of the numbers from six. (See 1 below)
   2. Square each of the differences. (See 2 below)
   3. Sum these squares. (See 3 below)
   4. Divide this sum by the number of squared terms that were added. (See 4 below)

1	2	3	4
$6 - 2 = 4$	$4^2 = 16$	16	$82 \div 6 = 13.7$
$6 - 2 = 4$	$4^2 = 16$	16	
$6 - 3 = 3$	$3^2 = 9$	9	
$6 - 9 = -3$	$(-3)^2 = 9$	9	
$6 - 10 = -4$	$(-4)^2 = 16$	16	
$6 - 10 = -4$	$(-4)^2 = 16$	16	
		82	

For Set B, the special number is 5.7.

Compute this special number for Set C.

Can you see any relationship between these special numbers and the sets from which they were derived?

7. The special number computed for the set (2, 2, 3, 9, 10, 10) was 13.7. If 123 were added to each of the numbers in the original set then what would the special number be for this new set?

8. Imagine that you are given a box with six buttons in it. They are numbered 1, 1, 2, 6, 7, and 7. The "mean number" of this set is four. Now you are to imagine that you proceed according to instructions (a) and (b):

(a) Shake the box. Pick one button from the box. Record its number and then return the button to the box. Repeat this step so that you have two numbers recorded. Compute the mean of this set of two numbers.

(b) Pick a set of eight numbers by using the procedure in (a). Compute the mean of this set of eight numbers.

Which of these two computed means, (a) or (b), is likely to be close to four (the mean of the six numbers in the box) or would it make no difference?

APPENDIX E

STUDENT REACTION TEST

STUDENT REACTION

TO THE STUDENT:

PLEASE CHECK EACH OF THE FOLLOWING STATEMENTS BY DRAWING A CIRCLE AROUND YOUR SELECTED RESPONSE.

THE KEY TO THE RESPONSES IS:

SA	STRONGLY AGREE
A	AGREE
U	UNDECIDED
D	DISAGREE
SD	STRONGLY DISAGREE

1. While studying the unit I enjoyed doing the experiments. SA A U D SD
2. While studying the unit I found that statistics made me feel uncomfortable and confused. SA A U D SD
3. While studying the unit I enjoyed drawing graphs to illustrate statistical concepts. SA A U D SD
4. While studying the unit I found that the activities were dull and boring. SA A U D SD
5. If I had time I would enjoy going beyond the assigned work in the unit. SA A U D SD
6. While studying the unit I had trouble with many of the terms and symbols in statistics. SA A U D SD
7. While studying the unit I found that most of the statistical ideas were understandable. SA A U D SD
8. While studying the unit I was able to do most of the exercises by myself. SA A U D SD

9. While studying the unit I felt that many of the lessons took too much time to understand thoroughly. SA A U D SD

10. While studying the unit I had trouble with most of the lessons. SA A U D SD

11. While studying the unit I found that statistics is more interesting than, or at least as interesting as, other mathematics I have studied so far. SA A U D SD

12. After studying the unit I found that I am interested in the ideas of statistics in that it does not deal with exact answers for a problem, instead it deals with the information of a set of scores as a whole and reasonable estimates for problem solutions. SA A U D SD

13. While studying the unit I often discussed statistical ideas with my friends outside the classroom. SA A U D SD

14. While studying the unit I did talk about the unit to my parents several times. SA A U D SD

15. After studying the unit I am interested in acquiring further statistical knowledge. SA A U D SD

16. After studying the unit I feel that I have learned some new terms and a statistical vocabulary. SA A U D SD

17. After studying the unit I feel that I have learned some new formulas that I did not know before. SA A U D SD

18.	After studying the unit I now understand how the information for a population can be estimated by the information from a sample.	SA	A	U	D	SD
19.	Doing experiments, drawing graphs, answering questions, and doing exercises while studying the unit really helped me to gain new knowledge of statistics.	SA	A	U	D	SD
20.	After studying the unit I feel that I have learned much about statistics that I did not know before.	SA	A	U	D	SD
21.	After studying the unit I see little use for this type of mathematics.	SA	A	U	D	SD
22.	Statistical knowledge gained from the unit will help me to understand the statistical information in newspapers and magazines.	SA	A	U	D	SD
23.	After studying the unit I still can not see the importance of statistics in everyday life.	SA	A	U	D	SD
24.	I think that the statistical knowledge gained from the unit is also useful for further studies.	SA	A	U	D	SD
25.	I believe that statistics could be useful for various future occupations.	SA	A	U	D	SD

26. Comments ....

....

APPENDIX F

TEACHER REACTION TEST

TEACHER REACTION

Based on your own assessment about the instructional unit "Basic Inferential Statistics," please give your opinions on the following five question areas:

1. What perceptions do you have of how well the unit "Basic Inferential Statistics" serves the general five objectives listed below?
    a) To develop an understanding of mathematical concepts and appreciation of mathematical structure.
    b) To develop skill in the use of the fundamental process.
    c) To develop systematic methods of analyzing problems and of presenting their solutions.
    d) To develop habits of precise thought and expression.
    e) To develop an understanding of the significance and application of mathematics in the modern world.
2. What perceptions do you have of the instructional suitability of the unit?
    - Instructional approach: using examples along with graphical illustrations, and doing experiments where appropriate.
    - The amount of work, time, money (if any), and the degree of difficulty on: lesson preparation, activity development, material management, extra help to students, class organization, and others.
    - The adequacy and student interest in materials used: established data in a box and personal data.
    - Other.
3. What perceptions do you have of the demands made on the teacher's subject background in teaching the unit?
    - Do you feel you have sufficient subject background to teach the unit?
    - Do you feel, in general, that junior high school mathematics teachers have sufficient subject background to teach the

unit?

- Other.

4. What perceptions do you have of the student reactions to the unit?

   - Do the students enjoy the activities?
   - Do the students have a lot of difficulty in understanding?
   - Do students express interest in the content?
   - Do the students feel they have gained new knowledge?
   - Do the students perceive the usefulness of the subject?

5. Should the unit or this type of mathematics be included in the core program?

   - If "no," why not?
   - If "yes".
     - What would be the suitable grade?
     - What topic should it replace?
     - Other.

APPENDIX G

THE STUDENTS' BACKGROUND

THE STUDENTS' BACKGROUND

Student*	I.Q. Scores**	Prev. Term Math Grade (%)	Age (months) (April '76)	Attendance (No of periods absent)
01A0	112	90	181	0
02A0	82	35	183	2
03A1	85	60	182	0
04A0	106	50	174	1
05A0	91	40	178	0
06A0	100	50	172	1
07A1	117	85	181	1
08A0	124	85	175	3
09A1	98	75	178	0
10A1	108	90	178	0
11A0	135	80	178	2
12A0	127	95	173	0
13A0	120	85	176	0
14A1	107	70	182	1
15A0	77	40	186	0
16A0	101	80	179	3
17A1	116	80	170	0
18A1	118	85	185	1
19A1	123	85	181	0
20A1	96	60	177	1
21A1	113	75	171	0
$\bar{x}$	107.43	71.19	178.10	0.76
SD	15.51	18.57	4.48	1.00

*Students' I.D.: First two digits - identified numbers given.
Letters: A - School A
B - School B
Last digit: 0 - a girl
1 - a boy.

**I.Q. scores from both schools were measured by using Lorge Thorndike Intelligence Test, November 1974.

Student	I.Q. Scores	Prev. Term Math Grade (%)	Age (months) (April '76)	Attendance (No of Periods Absent)
01B1	104	55	182	0
02B1	102	80	174	1
03B0	81	50	174	0
04B1	102	45	182	0
05B0	85	10	185	2
06B1	131	70	179	3
07B1	104	50	188	1
08B1	85	50	176	0
09B0	99	30	170	4
10B1	88	50	197	1
11B1	104	65	174	1
12B1	102	65	174	3
13B1	111	70	176	1
14B0	89	50	178	2
15B1	104	55	175	0
16B1	101	25	176	1
17B1	119	80	170	0
18B0	104	85	177	0
19B1	97	60	177	1
20B1	112	85	178	0
21B1	122	60	180	0
22B1	103	60	178	3
23B1	92	30	182	3
24B1	113	35	182	0
25B0	84	35	180	8
26B1	97	20	179	2
27B1	95	50	180	0
$\bar{x}$	101.11	52.59	178.63	1.37
SD	12.01	19.68	5.52	1.80

APPENDIX H

RESULTS OF SKILL TEST

Results of Skill Test

Student I.D.	Addition		Subtraction		Multiplication		Division		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
01A0	10	12	12	12	12	12	9	12	43	48
02A0	5	Abs.	3	Abs.	3	Abs.	3	Abs.	14	Abs.
03A1	4	6	5	6	4	6	5	8	18	26
04A0	9	10	8	10	11	11	10	10	38	41
05A0	5	4	5	5	4	4	3	2	17	15
06A0	7	6	7	7	8	10	7	8	29	31
07A1	12	11	12	12	11	12	11	12	46	47
08A0	11	10	11	11	12	11	12	12	46	44
09A1	7	7	4	10	4	9	7	9	22	35
10A1	12	11	10	11	12	12	11	11	45	45
11A0	11	12	12	12	10	12	11	9	44	45
12A0	12	12	12	12	12	12	12	12	48	48
13A0	11	9	7	8	11	10	10	12	39	39
14A1	9	10	9	9	10	9	9	8	37	36
15A0	6	7	7	8	7	6	5	6	25	27
16A0	8	6	7	8	7	10	7	9	29	33
17A1	12	12	12	12	10	12	8	10	42	46
18A1	12	12	12	12	12	10	12	12	48	46
19A1	10	11	10	11	8	12	8	11	36	45
20A1	5	4	4	3	4	7	4	6	17	20
21A1	6	10	7	10	5	12	8	12	26	44
$\bar{x}$	8.76	9.10	8.38	9.45	8.43	9.95	8.19	9.55	33.76	38.05
SD	2.81	2.77	3.09	2.65	3.28	2.44	2.93	2.68	11.56	9.85

Student I.D.	Addition		Subtraction		Multiplication		Division		Total	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
01B1	8	9	6	7	8	10	8	11	30	37
02B1	12	12	9	10	11	11	12	12	44	45
03B0	9	10	8	8	10	10	6	11	33	39
04B1	9	11	5	7	7	10	8	10	29	38
05B0	4	5	4	6	4	5	3	3	15	19
06B1	11	11	9	11	10	12	11	11	41	45
07B1	7	9	7	5	8	9	7	7	29	30
08B1	7	12	10	12	9	10	6	10	32	44
09B0	5	5	6	7	9	9	6	9	26	30
10B1	11	11	10	9	11	12	11	11	43	43
11B1	9	11	8	10	11	10	10	12	38	43
12B1	9	9	10	9	9	10	10	11	38	39
13B1	10	12	12	11	10	11	11	10	43	44
14B0	9	6	10	8	11	9	10	9	40	32
15B1	10	11	8	10	9	10	6	8	33	39
16B1	6	4	5	7	7	9	7	8	25	28
17B1	11	11	11	11	11	10	11	10	44	42
18B0	11	12	11	11	12	11	11	12	45	46
19B1	11	8	11	10	10	9	9	10	41	37
20B1	10	12	11	10	12	11	12	11	45	44
21B1	10	12	10	11	10	10	10	11	40	44
22B1	11	8	8	8	10	10	8	9	37	35
23B1	6	Abs.	8	Abs.	9	Abs.	7	Abs.	30	Abs.
24B1	8	11	11	11	12	12	8	10	39	44
25B0	8	Abs.	7	Abs.	8	Abs.	5	Abs.	28	Abs.
26B1	5	5	5	3	7	8	6	7	23	23
27B1	11	8	9	8	11	11	10	8	41	35
$\overline{x}$	8.81	9.40	8.48	8.80	9.48	9.96	8.48	9.64	35.26	37.80
SD	2.20	2.61	2.24	2.22	1.87	1.46	2.39	2.02	7.84	7.30

APPENDIX I

RESULTS OF ACHIEVEMENT TEST

ITEM ANALYSIS OF ACHIEVEMENT TEST PART I *

	1			2				3			4	5	6	7			8	9	10
Student I.D.	1.1	1.2	1.3	2.1	2.2	2.3	2.4	3.1	3.2	3.3				7.1	7.2	7.3			
01A0							✓					✓							
02A0		✓		✓			✓			✓	✓	✓	✓	✓	✓	✓	✓		✓
03A1							✓					✓						✓	
04A0				✓			✓		✓			✓							✓
05A0	✓			✓	✓		✓	✓	✓			✓	✓	✓			✓	✓	✓
06A0				✓			✓					✓						✓	✓
07A1				✓			✓										✓		
08A0												✓					✓		
09A1						✓						✓	✓		✓	✓	✓	✓	✓
10A1							✓												
11A0							✓					✓					✓		
12A0							✓					✓							
13A0							✓										✓		
14A1							✓					✓	✓				✓	✓	
15A0				✓	✓	✓	✓					✓	✓	✓		✓	✓	✓	✓
16A0							✓					✓						✓	
17A1				✓			✓												
18A1							✓					✓					✓		
19A1							✓					✓						✓	
20A1	✓		✓	✓			✓			✓		✓						✓	✓
21A1							✓					✓						✓	✓
Total: Wrong	2	1	1	8	2	2	19	1	2	2	1	17	5	3	2	4	12	9	7
Right	19	20	20	13	19	19	2	20	19	19	20	4	16	18	19	17	9	12	14

	11						12			13		14		15						16				17		
Student I.D.	11.1a	11.1b	11.2a	11.2b	11.3a	11.3b	12.1	12.2	12.3	13.1	13.2	14.1	14.2	15.1a	15.1b	15.2a	15.2b	15.3	15.4	16.1	16.2	16.3	16.4	17.1	17.2	17.3
01A0		✓				✓																				
02A0		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓			
03A1		✓				✓			✓					✓	✓	✓	✓			✓	✓		✓			
04A0	✓	✓	✓	✓	✓	✓						✓	✓				✓			✓			✓			
05A0	✓	✓		✓	✓	✓	✓	✓	✓			✓	✓	✓	✓		✓			✓	✓	✓	✓			✓
06A0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
07A1		✓				✓							✓			✓										
08A0		✓				✓																				
09A1		✓				✓				✓	✓	✓	✓	✓	✓	✓	✓				✓					
10A1				✓		✓										✓			✓	✓	✓					
11A0		✓				✓																				
12A0						✓																				
13A0													✓	✓												
14A1		✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓			✓	✓	✓				
15A0		✓		✓	✓	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓	✓	✓	✓	✓
16A0		✓	✓		✓	✓																	✓			
17A1		✓		✓	✓	✓						✓	✓													
18A1		✓				✓	✓	✓	✓						✓		✓	✓								✓
19A1					✓	✓									✓		✓									
20A1	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓			✓
21A1			✓		✓	✓				✓	✓				✓		✓			✓	✓					
Total: Wrong	4	17	4	8	11	20	6	6	7	6	6	9	11	8	11	9	12	5	4	10	7	6	8	2	2	4
Right	17	4	17	13	10	1	15	15	14	15	15	12	10	13	10	12	9	16	17	11	14	15	13	19	19	17

* ✓ represents test items with wrong answers.

ITEM ANALYSIS OF ACHIEVEMENT TEST PART I*

	1			2				3			4	5	6	7			8	9	10	11						12			13		14		15						16				17		
	1.1	1.2	1.3	2.1	2.2	2.3	2.4	3.1	3.2	3.3				7.1	7.2	7.3				11.1a	11.1b	11.2a	11.2b	11.3a	11.3b	12.1	12.2	12.3	13.1	13.2	14.1	14.2	15.1a	15.1b	15.2a	15.2b	15.3	15.4	16.1	16.2	16.3	16.4	17.1	17.2	17.3
01B1	✓			✓			✓					✓						✓			✓				✓						✓	✓				✓					✓		✓		
02B1					✓							✓								✓				✓	✓							✓													
03B0	✓								✓			✓					✓	✓	✓		✓		✓		✓	✓					✓	✓	✓	✓		✓	✓		✓	✓					
04B1	✓						✓					✓					✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓					
05B0				✓					✓		✓	✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓						✓	✓		✓		✓			✓	✓	✓				
06B1					✓							✓	✓						✓				✓		✓						✓	✓							✓						
07B1	Absent																																										Absent		
08B1				✓	✓		✓					✓					✓	✓		✓	✓			✓	✓						✓	✓	✓	✓	✓	✓		✓	✓	✓					
09B0	✓					✓					✓	✓	✓				✓	✓	✓		✓				✓						✓	✓								✓				✓	
10B1				✓			✓		✓			✓	✓	✓	✓		✓				✓				✓		✓				✓	✓	✓	✓	✓	✓	✓	✓			✓	✓			
11B1							✓					✓						✓			✓				✓	✓	✓	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓					
12B1	Absent																																										Absent		
13B1		✓																	✓				✓		✓													✓	✓	✓					
14B0	✓	✓	✓	✓	✓		✓					✓							✓		✓				✓		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					✓
15B1				✓								✓							✓		✓				✓						✓				✓	✓	✓	✓	✓	✓			✓		
16B1	✓	✓	✓	✓	✓				✓								✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓			✓	✓				✓		✓	✓	✓		✓			
17B1								✓	✓																						✓	✓	✓		✓										
18B0	✓						✓								✓													✓																	
19B1			✓									✓												✓	✓														✓	✓					
20B1																																													✓
21B1			✓	✓		✓	✓														✓		✓		✓						✓														
22B1	Absent																																										Absent		
23B1	✓	✓	✓	✓	✓		✓						✓				✓	✓	✓		✓				✓	✓	✓	✓			✓	✓	✓	✓	✓	✓		✓	✓				✓		
23B1				✓			✓		✓			✓							✓		✓				✓					✓			✓		✓		✓	✓	✓					✓	
24B1	✓		✓	✓		✓	✓	✓	✓			✓					✓		✓		✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓					✓
25B0	✓			✓		✓	✓		✓			✓						✓	✓	✓	✓			✓	✓						✓	✓	✓	✓	✓	✓			✓	✓					
26B1	✓			✓	✓	✓	✓					✓					✓	✓	✓	✓	✓	✓	✓	✓	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
27B1																																													
Total: Wrong	11	4	6	13	7	5	13	2	8	0	2	17	5	1	3	1	10	11	13	7	17	4	8	7	21	6	7	7	3	4	17	17	12	11	12	14	8	10	16	14	3	2	3	2	3
Right	13	20	18	11	17	19	11	22	16	24	22	7	19	23	21	23	14	13	11	17	7	20	16	17	3	18	17	17	21	20	7	7	12	13	12	10	16	14	8	10	21	22	21	22	21

*✓ represents test items with wrong answers.

RESULTS OF PART I, PART II, AND APPLICATION PART

Student	Part I	Part II	Application	Student	Part I	Part II	Application
01A0	41	14	18	01B1	33	10	12
02A0	13	2	2	02B1	39	8	17
03A1	32	7	10	03B0	27	10	9
04A0	29	10	13	04B1	20	1	7
05A0	15	6	7	05B0	23	3	10
06A0	15	1	1	06B1	36	10	15
07A1	38	12	16	07B1	Abs.	Abs.	Abs.
08A0	41	13	18	08B1	26	7	8
09A1	26	5	8	09B0	31	7	13
10A1	38	11	14	10B1	24	6	8
11A0	40	9	18	11B1	28	3	8
12A0	42	12	18	12B1	Abs.	Abs.	Abs.
13A0	40	13	15	13B1	38	11	15
14A1	25	6	8	14B0	22	3	7
15A0	15	1	2	15B1	32	10	10
16A0	37	11	16	16B1	21	5	10
17A1	37	11	16	17B1	39	11	14
18A1	33	11	14	18B0	41	9	18
19A1	38	13	16	19B1	39	10	16
20A1	15	3	5	20B1	44	10	17
21Ak	32	7	11	21B1	37	7	17
				22B1	Abs.	Abs.	Abs.
				23B1	21	2	8
				24B1	31	12	11
				25B0	18	3	6
				26B1	25	5	9
				27B1	18	6	5
				$\bar{x}$(A)	30.57	8.48	11.71
				SD(A)	10.29	4.23	5.73
				$\bar{x}$(B)	29.71	7.04	11.25
				SD(B)	8.01	3.30	3.99
				$\bar{x}$(AB)	30.11	7.71	11.47
				SD(AB)	9.05	3.79	4.83

APPENDIX J

RESULTS OF CONCEPTION TEST

RESULTS OF CONCEPTION TEST

Student I.D.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
01A0	(3,1)	(1,1)	(1,1)	(1,1)	(1.3,2.3)	(7,5)	(3,3)	(1.5,1.1)
02A0	(5,1)	(4,4)	(4,4)	(2,2)	(1.1,3)	(3,8)	(2,1)	(4,4)
03A1	--	--	--	--	--	--	--	--
04A0	--	--	--	--	--	--	--	--
05A0	(5,2)	(4,2)	(4,1)	(1,1)	(1.1,1.1)	(7,8)	(2,2)	(2.1,2.1)
06A0	--	--	--	--	--	--	--	--
07A1	(1,1)	(3,3)	(3,3)	(1,1)	(2.1,2.1)	(7,7)	(1,1)	(1.2,1.1)
08A0	(1,1)	(3,2)	(3,2)	(1,2)	(2.1,2.2)	(7,7)	(2,3)	(1.2,1.1)
09A1	(5,1)	(6,3)	(1,4)	(3,3)	(2.1,1.2)	(5,8)	(5,2)	(2.1,1.1)
10A1	(1,1)	(3,1)	(1,1)	(1,1)	(1.2,1.2)	(6,6)	(2,3)	(1.2,1.1)
11A0	(1,1)	(4,1)	(5,1)	(1,2)	(2.3,2.3)	(7,8)	(3,3)	(1.2,1.1)
12A0	(1,1)	(2,1)	(2,1)	(1,1)	(1.3,2.1)	(3,3)	(3,3)	(3.5,1.2)
13A0	(4,2)	(5,1)	(1,1)	(2,1)	(2.2,2.2)	(5,7)	(3,3)	(1.2,1.1)
14A1	(4,2)	(5,3)	(5,3)	(2,1)	(2.5,2.4)	(5,4)	(3,2)	(2.4,1.3)
15A0	(5,4)	(5,1)	(1,1)	(2,3)	(2.1,1.1)	(5,8)	(3,5)	(3.4,3.4)
16A0	(4,3)	(3,3)	(4,3)	(1,1)	(2.5,1.2)	(4,4)	(3,2)	(1.2,1.1)
17A1	(1,2)	(1,1)	(1,1)	(1,1)	(1.5,2.1)	(4,3)	(3,3)	(3.1,1.1)
18A1	(1,1)	(1,1)	(1,1)	(1,1)	(2.5,2.1)	(7,7)	(3,3)	(2.1,1.2)
19A1	(1,1)	(2,1)	(1,1)	(1,2)	(2.1,2.1)	(2,2)	(1,1)	(1.2,1.1)
20A1	--	--	--	--	--	--	--	--
21A1	(1,2)	(1,3)	(1,3)	(2,2)	(2.1,2.5)	(6,4)	(3,1)	(1.4,1.1)

Note:

1. Q1, Q2 . . . Q8 refer to Question number 1, 2, . . . 8 as illustrated in Appendix D.
2. List of students' responses to each question is illustrated in Chapter 6, Tables 6.7-6.14.
3. (m,n): m refers to item number in pretest response, n in post-test as appeared in Tables 6.7-6.14.

Student I.D.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
01B1	(1,1)	(5,3)	(4,3)	(1,1)	(2.1,1.2)	(7,7)	(3,4)	(3.1,1.1)
02B1	(1,1)	(3,4)	(3,3)	(1,1)	(1.1,1.5)	(7,8)	(4,4)	(3.1,2.1)
03B0	--	--	--	--	--	--	--	--
04B1	(2,2)	(4,3)	(1,3)	(3,4)	(1.1,1.2)	(7,8)	(2,6)	(1.2,1.1)
05B0	(4,1)	(4,5)	(5,6)	(3,4)	(1.1,2.1)	(7,8)	(2,1)	(1.2,4)
06B1	(1,1)	(1,1)	(1,1)	(1,1)	(1.3,1.2)	(6,2)	(3,3)	(3.1,1.2)
07B1	(1,1)	(4,4)	(4,3)	(2,1)	(1.3,1.1)	(7,7)	(2,6)	(2.3,2.1)
08B1	(4,2)	(4,3)	(4,3)	(1,1)	(1.2,1.2)	(7,7)	(2,6)	(2.2,1.3)
09B0	(5,1)	(6,2)	(6,1)	(3,1)	(1.1,1.1)	(7,7)	(2,6)	(1.3,1.3)
10B1	(4,5)	(1,3)	(1,3)	(2,2)	(1.1,1.2)	(7,7)	(2,1)	(3.3,1.3)
11B1	(1,3)	(3,3)	(3,3)	(1,4)	(1.3,2.1)	(7,8)	(2,3)	(2.3,3.1)
12B1	(1,1)	(3,1)	(3,1)	(1,1)	(1.1,1.1)	(5,5)	(2,3)	(2.1,2.2)
13B1	(1,1)	(3,1)	(4,1)	(3,3)	(2.1,1.4)	(7,6)	(2,3)	(3.1,1.1)
14B0	(5,1)	(6,4)	(6,4)	(1,2)	(1.1,1.1)	(7,8)	(2,6)	(1.3,2.1)
15B1	(4,1)	(4,1)	(3,1)	(1,1)	(1.6,1.2)	(7,7)	(2,3)	(3.1,1.3)
16B1	(1,2)	(4,4)	(1,3)	(1,1)	(1.1,2.1)	(7,7)	(2,5)	(2.1,2.1)
17B1	(2,2)	(5,2)	(5,2)	(1,1)	(1.3,1.4)	(7,8)	(2,1)	(3.2,1.1)
18B0	(1,1)	(4,1)	(4,1)	(1,1)	(1.5,1.2)	(5,5)	(2,3)	(3.1,1.1)
19B1	(1,1)	(1,1)	(1,1)	(1,1)	(1.3,2.2)	(1,1)	(3,1)	(1.2,1.1)
20B1	(1,1)	(1,1)	(1,1)	(1,2)	(1.3,1.2)	(3,2)	(3,1)	(2.1,1.1)
21B1	(1,1)	(3,1)	(3,1)	(1,1)	(1.1,2.1)	(7,5)	(2,3)	(1.2,1.2)
22B1	(2,1)	(4,3)	(4,4)	(1,1)	(2.2,2.2)	(7,8)	(2,3)	(2.1,1.1)
23B1	(5,5)	(3,1)	(3,6)	(1,2)	(1.2,1.1)	(7,7)	(2,5)	(1.2,1.3)
24B1	(1,1)	(3,1)	(1,1)	(2,1)	(1.2,2.1)	(7,7)	(3,3)	(1.2,1.2)
25B0	(1,2)	(6,1)	(1,1)	(1,2)	(2.5,1.1)	(7,7)	(3,3)	(1.2,1.3)
26B1	--	--	--	--	--	--	--	--
27B1	(3,3)	(4,3)	(4,1)	(1,2)	(1.1,1.2)	(4,4)	(6,4)	(2.3,1.3)

Note: 1. Q1, Q2 . . . Q8 refer to Question number 1, 2, . . . 8 as illustrated in Appendix D.

2. List of students' responses to each question is illustrated in Chapter 6, Tables 6.7-6.14.

3. (m,n): m refers to item number in pretest response, n in post-test as appeared in Tables 6.7-6.14.

APPENDIX K

RESULTS OF STUDENT REACTION TEST

RESULTS OF STUDENT REACTION TEST

Variable	Question No.	1			2			3			4			5		
		School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools
Enjoyment	1	0	6	6	2	7	9	4	4	8	10	8	18	5	2	7
	2	2	5	7	3	12	15	2	5	7	10	2	12	4	3	7
	3	0	4	4	1	4	5	4	8	12	10	8	18	6	3	9
	4	1	8	9	1	11	12	4	3	7	9	3	12	6	2	8
	5	2	14	16	5	6	11	6	7	13	8	0	8	0	0	0
	Total 1-5	5	37	42	12	40	52	20	27	47	47	21	68	21	10	31
Difficulty	6	0	6	6	7	6	13	1	5	6	11	9	20	2	1	3
	7	0	0	0	2	7	9	4	3	7	14	15	29	1	2	3
	8	0	0	0	1	6	7	2	1	3	16	19	35	2	1	3
	9	2	3	5	4	9	13	5	7	12	10	7	17	0	1	1
	10	0	0	0	3	7	10	3	8	11	15	11	26	0	1	1
	Total 6-10	2	9	11	17	35	52	15	24	39	66	61	127	5	6	11
Interest	11	2	4	6	3	7	10	4	6	10	8	7	15	4	3	7
	12	2	4	6	2	6	8	3	8	11	12	8	20	2	1	3
	13	1	10	11	11	12	23	2	1	3	6	2	8	1	2	3
	14	0	11	11	6	10	16	7	2	9	8	3	11	0	1	1
	15	1	9	10	3	11	14	8	3	11	9	2	11	0	2	2
	Total 11-15	6	38	44	25	46	71	24	20	44	43	22	65	7	9	16

Variable	Question No.	1			2			3			4			5		
		School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools	School A	School B	2 Schools
Learn new knowledge	16	0	0	0	0	0	0	1	0	1	15	24	39	5	3	8
	17	0	0	0	0	0	0	2	1	3	14	18	32	5	8	13
	18	0	1	1	2	2	4	1	5	6	16	15	31	2	4	6
	19	0	1	1	0	2	2	2	7	9	12	13	25	7	4	11
	20	0	0	0	0	2	2	1	2	3	15	17	32	5	6	11
	Total 16-20	0	2	2	2	6	8	7	15	22	72	87	159	24	25	49
Usefulness	21	1	6	7	1	7	8	6	11	17	8	3	11	5	0	5
	22	0	2	2	0	6	6	6	9	15	13	9	22	2	1	3
	23	1	4	5	3	9	12	2	7	9	10	5	15	5	2	7
	24	0	1	1	2	6	8	6	8	14	10	10	20	3	2	5
	25	2	3	5	0	2	2	3	6	9	13	13	26	3	3	6
	Total 21-25	4	16	20	6	30	36	23	41	64	54	40	96	18	8	26
5 variables combined 1-25		17	102	119	62	157	219	89	127	216	282	231	513	75	58	133

STUDENT COMMENTS

School A

1. This was a very good course I enjoyed taking it.
2. I enjoyed doing the graphs and like the whole course.
3. It is very educational for older classes.
4. I really like the course. It taught me something that I did not know and helped me to understand how statistics can be used and ran in everyday life.
5. Could of covered ideas slower to understand enjoyed it.
6. If we had more time with it we would understand and enjoy it more.
7. I felt that if we had more time we could have gone through the unit more slowly may be appreciating it a little more.
8. I liked this unit, it was kind of fun. I didn't like taking that much homework home, but I survived.
9. Could use more time to learn the work better.
10. I think we should have more time to learn statistics a little better.
11. Rush too fast at the end of the unit.
12. It wasn't a bad class to take, I didn't quite understand SD.
13. I did enjoy statistics but although sometimes didn't understand.
14. I did not get very much out of this unit. It went too fast for me to comprehend and I felt it was very confusing.
15. I thought it was kind of sick and you went too fast at the end.

School B

1. I liked it very much it was fun doing it.
2. I enjoyed these lessons because it was fun and worthwhile.
3. I wouldn't consider this line of work but it wasn't bad.
4. Some definitions were rather hard to learn and understand. After a few weeks I found it was quite boring, but quite enjoyable at a few points.

5. It's nice to learn, but I wouldn't want my work to have anything to do with it.

6. I did enjoy doing the new class. But as any new thing to accomplish, it was somewhat strange and difficult.

7. We took too much time on some lesson which made us rush through the last few lessons.

8-9. I didn't like the homework.

10. This course was fun at first and then started getting more complicated.

11. This lesson I found rather dull through most of it, but some, nice

12. Things, like standard deviation, were not made clear as to the value.

13. I did not like the lessons.

14-17. Boring!

APPENDIX L

RESULTS OF TEACHER REACTION TEST

RESULTS OF TEACHER REACTION TEST

Teacher A

1. These objectives are fulfilled but at varying degree. This is dependent primarily on student ability, student interest in math area, and the depth of math background.
2. Materials are satisfactory. There is practical and theoretical applications involved. Time allotment for such a course was insufficient.
3. To be a success in any field of instruction, a teacher must have some background in the subject area.

   (In general) This varies with different teachers.

   I feel the teacher should have some background in the field enough to explain any problems that might arise.
4. My present class was easily motivated with the exception of the students that had a weak background in math skills.

   Interest was great. Students did work on their own time at home, some stayed after school hours and questioned me about various areas taken.
5. It could be but it is dependent upon the student ability. For those weak in math skills a limited portion of this course could be applied only. The depth & content of material are dependent on the teacher's background, & also on the type of students. Graphs, histograms, figures, polygons could easily be adapted to all students' levels and ability, but some parts are not.

Teacher B

1. Listed in order of how I thought they were served best:

   a, c, d, e, b.

2. -Instructional approach was good. Time permitting I'd like to see more experiments and activities performed by the students (esp. on the lesson devoted to standard deviation).

   -As it was set up the amount of preparation time needed (by the teacher) was quite minimal; however if there were more activities involved this would be increased as a result of allowing time to mark and record. Also I think about every 5th lesson there could be included a review of the material covered up to that time.

   -The teacher had to be prepared enough to teach the basic ideas in each lesson before the students were asked to do the exercises. The instructional unit was not written in detail enough to be able to hand it out and let the students work on their own. As it was, even after going through most of the ideas with the class there were always some who had difficulties when doing their individual work.

   -They liked using their personal data.

3. I had previously taken an elementary statistics course (255) at U. of A. so I had sufficient background in this subject. I think this is a required course if you major in math, therefore all junior high math teachers should have sufficient background knowledge.

4. Overall I think the students enjoyed the activities. They had

some difficulty in understanding and seeing the usefulness of some of the concepts. Terms and concepts were often not clearly outlined for them and this tends to make them a little uneasy. I feel that at a J.H. level these must be defined quite clearly. Where the unit left off, I don't feel that most of the students perceived the usefulness of this subject. Maybe a lesson or 2 could be added to give them practical examples of how these ideas are used in life situations.

5. As the core program now stands there is no extra time to introduce this type of math at the Gr. IX level; however if there was time I think it would be an excellent supplementary or enrichment topic to cover. It would be suitable for Gr. 8 or 9.